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NUMERICAL SIMULATION OF NONLINEAR PHASE CONJUGATION PROCESS

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Abstract. This paper is devoted to the application of adaptable difference schemes for the simulation of propagation and interaction of focused laser beams. New effective processes for the solution of stationary nonlinear optics problems are proposed. Simulation results on real experiment data show the advantages of such schemes and iteration processes.

Key words: nonlinear optic, numerical simulation, adaptable difference schemes.

Introduction. A new method of theoretical study–computational experiment–is more and more often applied in various fields of nature sciences, usually simultaneously with extensive physical experiments. The present work treats the numerical aspects of computational experiment in nonlinear optic. It should be noted, that the numerical simulation in this field of physics is used rather widely and a reliability of computational experiments results is guaranteed by the existence of mathematical models, that adequately describe a real nonlinear optical process. It is well known, that extensive experiments with powerful lasers is a very complicated and often expensive task, because we have to deal with extremely fast processes, giant power of laser's beam, overcome a difficulty of registration of intermediate experiments results. The method of computational experiment gives us an opportunity to simulate all the processes thoroughly and to do a preliminary optimization of the experimental devices parameters, too. This not merely accelerates scientific investigations, but also helps to save energetic and material resources. A development of computational experiment method will depend heavily on the appearance of a new generation of computers, what in its turn will stimulate the evolution of the new even more sophisticated numerical methods.

The present paper deals with the two main problems. The first is the application of adaptable difference schemes for the simulation of propagation and interaction of focused laser beams. The second problem is the construction of effective interactive processes for the solution of stationary nonlinear optics problems, such as phase conjugation (optical wavefront reversal) in stimulated backscatter.

1. Equations. The dimensionless equations for the transient backward stimulated Brillouin scattering (SBS) are (Zel'dovich et al., 1985)

$$e_{L,S} = \frac{E_{L,S}}{E_0},\ \sigma = \frac{\sigma}{\sigma_0}, z' = \frac{z}{z_0},\ t = \frac{t}{t_0},\ t_0 = \frac{z_0}{V_L}$$

$$\frac{\delta e_L}{\delta t'} + \frac{\delta e_L}{\delta z'} - i\mu\Delta_\perp e_L = i\Gamma\sigma e_S, \tag{1.1}$$

$$\frac{\delta e_S}{\delta t'} + \frac{\delta e_S}{\delta z'} - i\mu\Delta_\perp e_S = i\Gamma\sigma^* e_L, \tag{1.2}$$

$$\frac{\delta\sigma}{\delta t} + a\sigma = i\Gamma_1 e_L e_S^* + \sigma_n\,, \tag{1.3}$$

where E_L, E_S - the complex pump (laser) and Stokes beams, respectively. In the case of cylindrically symmetric beams

$$\Delta_\perp e = \frac{1}{r}\frac{\delta}{\delta r} r \frac{\delta}{\delta r} u,\ 0 \leqslant r \leqslant R,\ 0 \leqslant z \leqslant L.$$

Boundary conditions are given separately at two end points of an interval

$$0 \leqslant z \leqslant L$$

$$e_L(0,r,t) = e_L^1(r,t),\ e_S(L,r,t) = e_S^1(r,t) \tag{1.4}$$

$$e_L(z,R,t) = 0,\ e_S(z,R,t) = 0, \tag{1.5}$$

$$r\frac{\delta}{\delta r} e_L(z,0,t) = 0,\ r\frac{\delta}{\delta r} e_S(z,0,t) = 0.$$

The appropriate initial conditions are given at $t = t_0$. Under steady state conditions, when $t \to \infty$ and

$$e_L^1(r,t) = e_L^1(r),\ e_S^1(r,t) = e_S^1(r),\ \sigma_n = 0,$$

the complex pump and SBS backscatter amplitudes satisfy the parabolic equations (Zel'dovich et al., 1985)

$$\frac{\delta e_L}{\delta z} - i\mu\Delta_\perp e_L = -G|e_S|^2 e_L, \tag{1.6}$$

$$\frac{\delta e_S}{\delta z} + i\mu\Delta_\perp e_S = -G|e_L|^2 e_S, \tag{1.7}$$

and boundary conditions

$$e_L(0,r) = e_L^1(r),\quad e_S(L,r) = e_S^1(r). \tag{1.8}$$

The separated boundary conditions (1.8) cause main difficulties for a numerical solution of the problem (1.6)–(1.8). In the case of focused Gaussian beams used as boundary conditions for $e_L(z,r,t)$, $e_S(z,r,t)$, the solution of a linear diffraction equation, given in dimensionless form

$$2i\frac{\delta u}{\delta z} + \Delta_{\perp} u = 0, \tag{1.9}$$

$$u(0,r) = \exp\left(-\frac{r^2}{2}\left(1 + i\frac{1}{f}\right)\right) \tag{1.10}$$

is a function

$$\begin{aligned} u(z,r) = &\frac{1}{1 - \frac{z}{f} + i\frac{z}{z_R}} \times \\ &\times \exp\left(\frac{-r^2}{2}\left(\frac{1}{w^2(z)} + i\frac{1}{R(z)}\right)\right). \end{aligned} \tag{1.11}$$

In this expression the spotsize $w(z)$, the radius of curvature $R(z)$ and the parameter Z_W are defined by

$$w^2(z) = \left(1 - \frac{z}{f}\right)^2 + z^2, R(z) = \frac{f - 2z + z/z_W}{1 - z/z_W}, z_W = \frac{f}{1 + f^2}.$$

If the lens focal length $f < L \ll 1$, then at $z = z_W$ the function $u(z,r)$ changes very rapidly and dense spatial mesh must be taken to convey the oscillations of the solution accurately. The difference schemes on adaptable spatial mesh are used in our paper to overcome such difficulties.

It is well known, that the conservation of energy quantities is very important for nonlinear optic problems. Simulating the qualitative features of nonlinear evolution, equations is a desirable attribute of the discretization schemes.

Lemma 1.1. The solution of (1.6)–(1.8) satisfies the relation of energy conservation

$$\|e_L(z_1)\|^2 - \|e_S(z_1)\|^2 = \|e_L(z_2)\|^2 - \|e_S(z_2)\|^2, \tag{1.12}$$

$$0 \leqslant z_1 \leqslant z_2 \leqslant L,$$

where

$$\|u\|^2 = (u,u), \ (u,v) = \int\limits_0^R r u(r) v^*(r) dr.$$

For the proof of lemma it is sufficient to multiply the equations (1.6)–(1.7) by $e_L^*(z,r)$, $e_S^*(z,r)$, respectively, and to add the real parts of the obtained equations.

Remark 1.1. The analogous results are true for the solution of transient problem (1.1)–(1.5) (Čiegis, 1987; 1988).

2. Difference scheme. We use a splitting method to construct the basic scheme

$$\frac{\widetilde{u} - u(-1)}{\tau} - i\mu\Lambda\frac{\widetilde{u} + u(-1)}{2} = 0, \tag{2.1}$$

$$\frac{\sigma^p - \sigma}{\tau} + a\frac{\sigma^p + \sigma}{2} = i\Gamma\widetilde{u}v^*, \tag{2.2}$$

$$\frac{\widehat{u} - \widetilde{u}}{\tau} = i\Gamma\bar{\sigma}^p\bar{v}, \frac{\widetilde{v} - v}{\tau} = i\Gamma\bar{\sigma}^{p*}\bar{u}, \tag{2.3}$$

$$\bar{\sigma}^p = 0,5(\sigma^p + \sigma), \ \bar{u} = (\widehat{u} + \widetilde{u})0,5, \bar{v} = 0,5(\widetilde{v} + v),$$

$$\frac{\widehat{\sigma} - \sigma}{\tau} + a\frac{\widehat{\sigma} + \sigma}{2} = i\Gamma\bar{u}\bar{v}^*, \tag{2.4}$$

$$\frac{\widehat{v}(-1) - \widetilde{v}}{\tau} - i\mu\Lambda\frac{\widehat{v}(-1) + \widetilde{v}}{2} = 0. \tag{2.5}$$

The notations and conventions here are adopted as introduced by Samarskij (1983)

$$\Lambda u = \frac{1}{r}(\bar{r}u_{\bar{r}})_r, \ u = u(z_j, r_k, t_n),$$
$$\widehat{u} = u(z_j, r_k, t_{n+1}), \ u(-1) = u(z_{j-1}, r_k, t_n),$$
$$z_j = z_{j-1} + \tau, \ t_n = t_{n-1} + \tau, \ r_k = r_{k-1} + h.$$

Theorem 2.1. The scheme (2.1)–(2.5) preserves the conservation of energy

$$\|\widehat{u}\|^2 + \|\widehat{v}(-1)\|^2 = \|u(-1)\|^2 + \|v\|^2, \tag{2.6}$$

the solution of the difference problem (2.1)–(2.5) converges to a solution of the problem (1.1)–(1.5) in L_2 norm as $O(\tau + h^2 \ln h)$.

Proof. Computing the inner product of both sides of (2.3) with $\bar{u}^*$, $\bar{v}^*$, respectively, summing up for r_i and taking the real part, we have

$$\|\widehat{u}\|^2 + \|\widetilde{v}\|^2 = \|\widetilde{u}\|^2 + \|v\|^2 \tag{2.7}$$

Analogously, from the equations (2.1), (2.5) it follows, that

$$\|\widetilde{u}\|^2 = \|u(-1)\|^2, \ \|\widehat{v}(-1)\|^2 = \|\widetilde{v}\|^2.$$

By the above equalities and from (2.7), we have (2.6). The proof that the solution of the difference problem (2.1)–(2.5) converges to the solution of the problem (1.1)–(1.5) is entirely similar to a method used by Čiegis (1988).

Remark 2.1. The implementation of this scheme is non-iterative and only the tridiagonal systems of equations are required to be solved at each time step.

The following difference scheme is proposed to solve the numerical problem (1.6)–(1.8)

$$\frac{\widetilde{u} - u}{\tau} - i\mu\Lambda\frac{\widetilde{u} + u}{2} = 0, \tag{2.8}$$

$$\frac{\widetilde{v} - v}{\tau} + i\mu\Lambda\frac{\widetilde{v} + v}{2} = 0, \tag{2.9}$$

$$\frac{\widehat{u} - \widetilde{u}}{\tau} = -G\left|\frac{\widehat{v} + \widetilde{v}}{2}\right|^2 \frac{\widehat{u} + \widetilde{u}}{2}, \tag{2.10}$$

$$\frac{\widehat{v} - \widetilde{v}}{\tau} = -G\left|\frac{\widehat{u} + \widetilde{u}}{2}\right|^2 \frac{\widehat{v} + \widetilde{v}}{2}. \tag{2.11}$$

Theorem 2.2. The difference scheme (2.8)–(2.11) possesses the main conservation quantity

$$||\widehat{u}||^2 - ||\widehat{v}||^2 = ||u||^2 - ||v||^2, \qquad (2.12)$$

where

$$||u||^2 = (u,u)_h,\ (u,v)_h = \sum_{j=1}^{N} r_j u_j v_j^* h.$$

Proof. Computing the product of both sides of (2.10), (2.11) with $(\widehat{u}+\widetilde{u})^*$, $(\widehat{v}+\widetilde{v})^*$, respectively, subtracting the obtained equalities and taking the real part, we have equalities

$$|\widehat{u}_j|^2 - |\widehat{v}_j|^2 = |\widetilde{u}_j|^2 - |\widetilde{v}_j|^2,\ j = 1,2,\ldots,N. \qquad (2.13)$$

Summing up (2.13) for $j = \overline{1,N}$,we get

$$||\widehat{u}||^2 - ||\widehat{v}||^2 = ||\widetilde{u}||^2 - ||\widetilde{v}||^2\ . \qquad (2.14)$$

From the above equalities and from (2.14),we obtain (2.12).

Remark 2.2. A simple Taylor expansion reveals that local truncation errors for the difference scheme (2.8)–(2.11) can be bounded by $C_1(\tau + h^2 \ln h)$.

3. The iterative procedure for (2.8)–(2.11). The finite difference scheme (2.8)–(2.11) gives us a nonlinear system of 2NM complex unknown quantities, where N, M–numbers of the grid points in r and z directions, respectively. In this paper we propose several iterative procedures for the solution of this problem.

Method 1. The difference scheme (2.8)–(2.11) is replaced by the time dependent equations (2.1)–(2.5) with boundary conditions (1.8). These equations are simply integrated by a non-iterative algorithm (2.1)–(2.5) over many optical transits across the medium until a steady state is finally attained.

We will approximately estimate the computing time required to attain the fixed accuracy by this method. The computing time is proportional to the number of elementary operations (addition, multiplication), used to solve the problem. The realization of one time step by a non-iterative finite difference scheme (2.1)–(2.5) requires $O(NM)$ operations, i.e., it is economical. But the number of iterations (time steps) is roughly estimated by $K = O(M)$. This follows from the fact, that time dependent equations (2.1)–(2.5) are hyperbolic, and the time required by the wave to go once the interval $O \leqslant z \leqslant L$ is equal to the length of this interval $T_1 = L$ (mesh size in time is equal to the mesh size in z direction for the finite difference scheme (2.1)–(2.5)). So, the amount of computer operations (computation time) for Method 1 is

$$Q_1 = KQ_0 = CNM^2 \tag{3.1}$$

and it is too large to use this method in computational experiments. The results of numerical experiments confirm the validity of estimation (3.1).

Method 2. We propose to replace the problem (1.6)–(1.8) by time dependent equations

$$\frac{\delta^2 u}{\delta t \delta z} + \frac{\delta u}{\delta z} - i\mu\Delta_\perp u = -G\left|v\right|^2 u, \tag{3.2}$$

$$\frac{\delta^2 v}{\delta t \delta z} + \frac{\delta v}{\delta z} + i\mu\Delta_\perp v = -G\left|u\right|^2 v, \tag{3.3}$$

and to construct iteration procedures for the finite difference scheme (2.8)–(2.11) as time dependent difference schemes for the problem (3.2), (3.3). For example, we can get the iteration procedure

$$\frac{\widetilde{u}^s - u^s}{\tau} - \frac{\widetilde{u}^{s-1} - u^{s-1}}{\tau} + \\ +\gamma\left(\frac{\widetilde{u}^{s-1} - u^{s-1}}{\tau} - i\mu\Delta_\perp\frac{\widetilde{u}^s + u^s}{2}\right) = 0, \tag{3.4}$$

$$\frac{\widehat{u}^s - \widetilde{u}^s}{\tau} - \frac{\widehat{u}^{s-1} - \widetilde{u}^{s-1}}{\tau} + \\ +\gamma\left(\frac{\widehat{u}^{s-1} - \widetilde{u}^{s-1}}{\tau} + G\left|\frac{\widehat{v}^{s-1} + \widetilde{v}^{s-1}}{2}\right|^2 \frac{\widehat{u}^s + \widetilde{u}^s}{2}\right) = 0, \quad (3.5)$$

$$\frac{\widehat{v}^s - \widetilde{v}^s}{\tau} - \frac{\widehat{v}^{s-1} - \widetilde{v}^{s-1}}{\tau} + \\ +\gamma\left(\frac{\widehat{v}^{s-1} - \widetilde{v}^{s-1}}{\tau} + G\left|\frac{\widehat{u}^s + \widetilde{u}^s}{2}\right|^2 \frac{\widehat{v}^s + \widetilde{v}^s}{2}\right) = 0, \quad (3.6)$$

$$\frac{\widetilde{v}^s - v^s}{\tau} - \frac{\widetilde{v}^{s-1} - v^{s-1}}{\tau} + \\ +\gamma\left(\frac{\widetilde{v}^{s-1} - v^{s-1}}{\tau} + i\mu\Delta_\perp \frac{\widetilde{v}^s + v^s}{2}\right) = 0, \quad (3.7)$$

where γ is the mesh size of fictitious time (iteration parameter).

Theorem 3.1. The iteration method (3.4)–(3.7) is convergent for $\gamma \leqslant \gamma_0, \quad G \leqslant G(\gamma)$.

The realization of one iteration step by (3.4)–(3.7) requires $Q_0 = Q(NM)$ operations, the total number of iterations required to attain the fixed accuracy is finite and does not depend on the mesh parameters h, τ, so the computation time for Method 2 is estimated by

$$Q_2 = KQ_0 = C_1C_2NM = C_3NM. \quad (3.8)$$

Remark 3.1. If we choose the iteration parameter $\gamma = 1$ in the equations (3.4)–(3.7), we get a simple iteration procedure, proposed by Čiegis, Norvaišas (1989). In that work it is proved that such iteration method is convergent only for the limited values of a coupling coefficient G.

Method 3. This method is a generalization of the iteration procedure, proposed by Čiegis et al., (1989, 1990)

$$\frac{\widetilde{u}^s - u^s}{\tau} - i\mu\Lambda\frac{\widetilde{u}^s + u^s}{2} = 0, \tag{3.9}$$

$$\frac{\widehat{u}^s - \widetilde{u}^s}{\tau} = -G\left|\frac{\widehat{v}^{s-1} + \widetilde{v}^{s-1}}{2}\right|^2 \frac{\widehat{u}^s + \widetilde{u}^s}{2}, \tag{3.10}$$

$$\frac{\widehat{v}^s - \widetilde{v}^s}{\tau} = -\gamma(\widehat{v}^s, \widetilde{v}^s)G\left|\frac{\widehat{u}^s + \widetilde{u}^s}{2}\right|^2 \frac{\widehat{v}^s + \widetilde{v}^s}{2}, \tag{3.11}$$

$$\gamma(\widehat{v}^s, \widetilde{v}^s) = \left|\widehat{v}^{s-1} + \widetilde{v}^{s-1}\right|^2 / \left|\widehat{v}^s + \widetilde{v}^s\right|^2, \tag{3.12}$$

$$\frac{\widetilde{v}^s - v^s}{\tau} + i\mu\Lambda\frac{\widetilde{v}^s + v^s}{2} = 0. \tag{3.13}$$

Theorem 3.2. The iteration procedure (3.9)–(3.13) is conservative, i.e., for all $s = 1, 2, \cdots$ solutions $\overset{s}{u}, \overset{s}{v}$ satisfy the energy conservation relation

$$\|\widehat{u}^s\| - \|\widehat{v}^s\|^2 = \|u^s\|^2 - \|v^s\|^2. \tag{3.14}$$

Proof. From equations (3.10)–(3.12) we obtain the equalities

$$\left|\widehat{u}_j^s\right|^2 - \left|\widehat{v}_j^s\right|^2 = \left|\widetilde{u}_j^s\right|^2 - \left|\widetilde{v}_j^s\right|^2.$$

The remained part of the proof coincides with the proof of Theorem 2.2.

The solution of nonlinear equation (3.11) could be found by the following procedure

$$Re\widetilde{v}^s = qRe\widehat{v}^s, \quad Im\widetilde{v}^s = qIm\widehat{v}^s, \tag{3.15}$$

$$\left|\widetilde{v}^s\right|^2 = \left|\widehat{u}^s\right|^2 - \left|\widetilde{u}^s\right|^2 + \left|\widehat{v}^s\right|^2$$

$$q^2 = \left(\left|\widehat{v}^s\right|^2 + \left|\widehat{u}^s\right|^2 - \left|\widetilde{u}^s\right|^2\right) / \left|\widehat{v}^s\right|^2. \tag{3.16}$$

The realization of one iteration step by Method 3 with such a procedure (3.15)–(3.16) requires $Q_0 = O(NM)$ operations. The convergence of iteration procedure is investigated by Čiegis and Norvaišas (1989).

Remark 3.2. Method 3 could be supplemented by an additional technique of stabilization, where the replacement is performed after solving (3.9)–(3.13)

$$v^s = \alpha v^s + (1-\alpha)v^{s-1},\ \widetilde{v}^s = \alpha\widetilde{v}^s + (1-\alpha)\widetilde{v}^{s-1},\ 0 < \alpha < 1.$$

Method 4. (shooting method). It is apparent from the equation (1.8), that some boundary values of $u(r,z), v(r,z)$ at $z = 0$ or $z = L$ are not given. To be able to solve the system (2.8)–(2.11) by the initial–value method,we have to guess the missing boundary conditions at the point $z = 0$. Taking these missing boundary–values as parameters q_j, we can write the boundary conditions

$$u(r_j, z=0) = e_L^1(r_j),\ v(r_j, z=0) = q_j. \tag{3.17}$$

Applying the initial–value method, the solutions $u(r_j, z_k, q_1, \cdots, q_N)$, $v(r_j, z_k, q_1, \cdots, q_N)$ may be found. By matching the solution $v(r_j, L, q_1, \cdots, q_N)$ at the point $z = L$, the following nonlinear algebraic equations

$$v(r_k, L, q_1, \cdots, q_N) = e_S{}^1(r_k),\ k = 1, \cdots, N \tag{3.18}$$

for $\overrightarrow{q} = (q_1, \cdots, q_N)$ are obtained. The problem is converted into solving the equations (3.18). The Newton–Raphson method can be used to find the roots of equations (3.18).

Remark 3.3. The computational time Method 4 is proportional to $O(N^2M + N^3)$, i.e., Method 4 is not economical.

Method 5. The Newton–Raphson method can be used to find the solution of the finite difference scheme (2.8)–(2.11)

$$\overset{s}{y} = \overset{s-1}{y} + \tau_s \overset{s}{p}, \quad \overset{s}{v} = \overset{s-1}{v} + \tau_s \overset{s}{w},$$

where $\overset{s}{p}$, $\overset{s}{w}$ are obtained from the linear problem

$$\frac{\widetilde{p}^s - p^s}{\tau} - i\mu\Lambda\frac{\widetilde{p}^s + p^s}{2} = 0, \tag{3.19}$$

$$\frac{\widetilde{w}^s - w^s}{\tau} + i\mu\Lambda\frac{\widetilde{w}^s + w^s}{2} = 0, \tag{3.20}$$

$$\begin{aligned}\frac{\widehat{p}^s - \widetilde{p}^s}{\tau} = &- G\Bigg(\left|\frac{\widehat{v}^{s-1} + \widetilde{v}^{s-1}}{2}\right|^2 \frac{\widehat{p}^s + \widetilde{p}^s}{2} + \\ &+ R_v\left(\frac{\widehat{v}^{s-1} + \widetilde{v}^{s-1}}{2}\right)\frac{\widehat{y}^{s-1} + \widetilde{y}^{s-1}}{2}\frac{\widehat{w}^s + \widetilde{w}^s}{2}\Bigg),\end{aligned} \tag{3.21}$$

$$\begin{aligned}\frac{\widehat{w}^s - \widetilde{w}^s}{\tau} = &- G\Bigg(\left|\frac{\widehat{y}^{s-1} + \widetilde{y}^{s-1}}{2}\right|^2 \frac{\widehat{w}^s + \widetilde{w}^s}{2} + \\ &+ R_v\left(\frac{\widehat{y}^{s-1} + \widetilde{y}^{s-1}}{2}\right)\frac{\widehat{v}^{s-1} + \widetilde{v}^{s-1}}{2}\frac{\widehat{p}^s + \widetilde{p}^s}{2}\Bigg),\end{aligned} \tag{3.22}$$

$$p^s(r_j, z = 0) = 0, \quad w^s(r_j, z = L) = 0, \tag{3.23}$$

$$R_v(u) = \frac{\delta |u|^2}{\delta u}.$$

The solution of a linear problem (3.19)–(3.23) could be found by the iteration procedure proposed in Method 2.

4. Adaptable difference schemes. The mesh transformation for the difference schemes (2.1)–(2.6), (2.8)–(2.11) is performed in the first step of splitting schemes, when the focused beams (1.10) are used as boundary conditions.

Method 1. The numerical solution of linear problems, corresponding to (2.1), (2.5), (2.8), (2.9), is based on the representation of $e(r,z)$ as a finite series

$$e(r,z) = \sum_{p=1}^{P} c_p(z_0) L_p(z,r), \tag{4.1}$$

where natural choice for the basis set L_p is the Laguerre–Gaussian functions, which satisfy the equation (1.9), (1.10) exactly. The basis set L_p is complete and orthogonal, so the coefficients c_p are obtained from the equality

$$c_p(z_l) = (e(r,z_l), L_p(z_l,r))/(L_p(z_l,r)). \tag{4.2}$$

Inner products then are approximated by sums with the local truncation error $O(h^2)$. The mesh $w_r(z_{l+1})$ is rescaled according to a focused beam's spotsize $w(z)$.

Remark 4.1. There is no fast transform algorithm for the numerical evaluation of (4.1), (4.2), so Method 1 is not economical if P is large.

Method 2. Less universal, but economical are adaptable difference schemes, where a coordinate transformation is related to the exact solution (1.11) of a linear problem for the propagation of focused laser beam (1.9), (1.10). It should be noted, that only a deviation from the exact solution (1.11) is computed numerically by a difference scheme. The simplest of such transformations is one originally due to Talanov (1970), more general transformation is proposed by Fleck et al. (1976).For $z \leqslant z_W$ we use the coordinate transformation

$$r' = r/(1 - z/z_L), \; z' = z/(1 - z/z_L), \tag{4.3}$$

$$u'(z',r') = u(z,r)\left(1 - \frac{z}{z_L}\right)\exp\left(i\frac{r^2}{2(z - z_L)}\right), \; z_L > z_W,$$

where $u'(z',r')$ satisfies the same free propagation equation (1.9), but the radius of curvature in (1.10) is replaced by

$$1/f' = 1/f - 1/z_L. \tag{4.4}$$

For $z > z_W$ we use a similar coordinate transformation

$$\begin{aligned} r'' &= r\Big/\left(1 - \frac{2z_W - z}{z_L}\right), \\ z'' &= (2z_W - z)\Big/\left(1 - \frac{2z_W - z}{z_L}\right), \\ u'' &= u(z,r)\left(1 - \frac{2z_W - z}{z_L}\right)\exp\left(-i\frac{r^2}{2(z_W - z - z_L)}\right), \end{aligned} \tag{4.5}$$

at $z = z_W$ we get an additional compatibility condition for $u''(z,r)$

$$u''(z = z_W, r') = u'(z = z_W, r')\exp\left(i\frac{r'^2}{z_L - z_W}\right). \tag{4.6}$$

Remark 4.1. The parameter z_L we propose to take such that the spotsize of transformed mesh at $z = z_W$ would be equal to the exact focused beam's spotsize $\omega(z_W)$

$$1 - z_W/z_L = \omega(z_W), \;\; z_L = f(1 + \omega(z_W)). \tag{4.7}$$

Remark 4.2. The nonlinear problem (1.6)–(1.8) remains the same after the Talanov's coordinate transformation, so this transformation was used for the numerical simulation of phase conjugation in stimulated backscatter.

Method 3. The coordinate transformation (4.3)–(4.6) is different for $z \leqslant z_W$ and for $z > z_W$ and this is not convenient for numerical simulation. The coordinate transformation

(4.8)–(4.9) has no such defect (see Ulrich, 1978)

$$r' = r/\omega(z), \; z' = \operatorname{arctg}\left(z - \frac{1 - z/f}{f}\right), \tag{4.8}$$

$$u'(r', z') = u(r, z)\omega(z)\exp\left(-i\frac{r^2}{2\omega(z)}\left(z - \frac{1 - z/f}{f}\right)\right),$$

where $u'(r', z')$ satisfies the parabolic equation

$$2i\frac{\delta u'}{\delta z'} + \nabla_\perp u' - r'^2 u'(r', z') = 0$$

with appropriate boundary condition.

The standard mesh interval (z_i, z_{i+1}) is divided into smaller intervals at the first splitting step, if we use mesh transformation, given by Methods 2 and 3 for a construction of the adaptable finite difference scheme on the basis of scheme (2.1)–(2.5). This procedure increases the accuracy of the difference schemes with a little increase of total computational time.

Some results of numerical experiment in which the linear problem (1.9), (1.10) was solved by a standard difference scheme (2.1) and adaptable difference schemes of Method 2 and Method 3 are given in Table 1. The parameters of the problem were

$$L = 0.1, \quad R = 3.5, \quad N = 60, \quad M = 50,$$

the global error $\varepsilon = \|u - y\|_c / (\varepsilon_0 + \|u\|_c)$ is outlined in Table 1.

Table 1 Results of numerical experiment

f	(2.1)	Method 2	Method 3
0.5	0.00132	0.00155	0.00219
0.2	0.03232	0.00161	0.00345
0.15	0.11035	0.00166	0.00500
0.05	2.02534	0.00363	0.00561

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