INFORMATICA, 2022, Vol. 33, No. 3, 437–476

DOI: https://doi.org/10.15388/21-INFOR465

A Novel T-Spherical Fuzzy REGIME Method for Managing Multiple-Criteria Choice Analysis Under Uncertain Circumstances

Ting-Yu CHEN

Graduate Institute of Management, Department of Industrial and Business Management, Chang Gung University, No. 259, Wenhua 1st Rd., Guishan District, Taoyuan City 33302, Taiwan
e-mail: tychen@mail.cgu.edu.tw

Received: June 2021; accepted: November 2021

Abstract. The theory of T-spherical fuzzy (T-SF) sets possesses remarkable capability to manage intricate uncertain information. The REGIME method is a well-established technique concerning discrete choice analysis. This paper comes up with a multiple-criteria choice analysis approach supported by the REGIME structure for manipulating T-SF uncertainties. This paper constructs new-created measurements such as superiority identifiers and guide indices for relative attractiveness and fittingness, respectively, between T-SF characteristics. This study evolves the T-SF REGIME I and II prioritization procedures for decision support. The application and comparative studies exhibit the effectiveness and favorable features of the propounded T-SF REGIME methodology in real decisions.
Key words: T-spherical fuzzy set, REGIME method, multiple-criteria choice analysis, superiority identifier, guide index.

1. Introduction

Uncertain decisions often take place in miscellaneous kinds of multiple criteria evaluation and assessment processes, especially in coping with complicated realistic problems (Dogan, 2021; Farrokhizadeh *et al.*, 2021). As the ever-increasing complexity of problems, uncertain decisions can be addressed by various fuzzy methods and techniques within indistinct and equivocal environments, and it can call for innovative high-order fuzzy approaches to generate interpretable solutions and efficacious decisions (Ashraf and Abdullah, 2021; Gül, 2021; Özlü and Karaaslan, 2021). In particular, the uncertain set of T-spherical fuzziness has been in a position of considerable influence for manipulating ambiguous and equivocal information in intricate real-world circumstances (Chen *et al.*, 2021; Guleria and Bajaj, 2021; Munir *et al.*, 2021; Özlü and Karaaslan, 2021). This section provides a concise review of high-order fuzzy approaches to multiple-criteria choice analysis. Special attention will be paid to a well-established qualitative evaluation method, named the REGIME method.

1.1. *T-Spherical Fuzziness with Decision-Making Applications*

Realistic decision-making activities are usually highly sophisticated and poorly structured, and many classical decision models cannot directly deal with these complicated problems (Alipour *et al.*, 2021; Garg, 2021b; Garg and Rani, 2021; Ullah *et al.*, 2021b). Numerous classical decision models manage crisp assessment data, which means that the subjective judgment offered by the decision maker is expressed as a precise number. Nevertheless, in considerable down-to-earth situations, the decision information may be inaccurate and/or imprecise (Alipour *et al.*, 2021; Garg, 2021a; Ullah *et al.*, 2021a; Wang and Chen, 2021). Moreover, the decision maker may be unable to explicitly give accurate numerical values for subjective evaluations in uncertain circumstances (Gao and Deng, 2021; Garg, 2021b; Garg and Rani, 2021). In particular, certain judgment criteria are qualitative or equivocal in nature, making it difficult for the decision maker to exploit precise values to externalize subjective assessments and preferences (Alipour *et al.*, 2021; Garg and Rani, 2021; Ullah *et al.*, 2021b). Convoluted decision-making issues usually involve inaccuracy, ambiguity, and indefiniteness, resulting in traditional decision models and relevant canonical techniques often ineffective when manipulating subjective assessment information (Garg, 2021a, 2021b; Wang and Chen, 2021). The aforementioned difficulties and considerations make the notion of fuzzy sets flourish in decision theory (Alipour *et al.*, 2021; Gao and Deng, 2021; Garg, 2021b).

There are different general variants of the fuzzy models that delineate an object's membership in a fuzzy set in various formats (Smarandache, 2019; Ullah *et al.*, 2020a, 2020b). The notion of ordinary fuzzy sets generalized classical sets and permits a gradual appraisal about an object's membership in a set. The real world would be full of indeterminism, vagueness, and limited knowledge; thus, the complexities associated with the high-order fuzziness are dependent in a sophisticated way on their uncertainty (Chen, 2021; Donyatalab *et al.*, 2020; Farrokhizadeh *et al.*, 2021; Gül, 2021). In such considerations, it is evidently meaningful to constitute non-standard fuzzy configurations for modelling imprecision and murkiness in recent decades, such as advanced fuzzy models regarding intuitionistic fuzziness (Atanassov, 1986), Pythagorean fuzziness (Yager, 2013), Fermatean fuzziness (Senapati and Yager, 2019a, 2019b), q-rung orthopair fuzziness (Yager, 2017), picture fuzziness (Cuong, 2014), spherical fuzziness (Gündoğdu and Kahraman, 2019), and T-spherical fuzziness (Mahmood *et al.*, 2019). As an efficacious means to expound ambiguous and equivocal information, the generalizations of fuzzy sets exhibit a mathematical strength to expatiate on the uncertainty in subjective thinking and cognitive processes for the reasoning behind intricate decisions in a logical and sensible way (Garg, 2021a; Senapati and Yager, 2019b; Shahzadi *et al.*, 2021; Ullah *et al.*, 2020a). After introducing the generalizations of fuzzy sets, the ordinary fuzzy version of decision models and techniques also began to attain full developments via these higher-order fuzzy sets. Especially, the configuration involving T-spherical fuzziness is a recent advancement in fuzzy theory and has a magnificent capability of tackling decision-making in multiple criteria choice issues (Chen *et al.*, 2021; Garg *et al.*, 2021; Wang and Chen, 2021; Zeng *et al.*, 2020).

The conceptual framework of T-spherical fuzzy (T-SF) sets, initially introduced by Mahmood *et al.* (2019), is a significant tool for decision makers with multiple criteria evaluation and assessment problems under complicated uncertain circumstances. The uncertain sets of intuitionistic, Pythagorean, Fermatean, and q-rung orthopair fuzziness are elucidated by virtue of belonging and non-belonging functions. More precisely, a total sum of both functions takes a value in a real unit interval from 0 to 1 in the intuitionistic fuzzy model (Atanassov, 1986); the square sum of both functions is valued in [0, 1] in the Pythagorean fuzzy model (Yager and Abbasov, 2013); the cube sum of both functions is valued in [0, 1] in the Fermatean fuzzy model (Senapati and Yager, 2019a, 2019b); the sum of both functions to the q-th power (q is a positive integer) is valued in [0, 1] predicated on the q-rung orthopair fuzzy model (Khan *et al.*, 2021a; Yager, 2017). Herein, the residual term (i.e. the length of the real unit interval excluding the belonging and non-belonging parts) is considered as the grade of indeterminacy. Especially, the conception of q-rung orthopair fuzziness can be deliberated as a broad-ranging kind of non-standard fuzzy models because it becomes the intuitionistic, Pythagorean, and Fermatean fuzzy models when $q = 1, 2, 3$, respectively (Akram *et al.*, 2021a, 2021b; Khan *et al.*, 2021a). The conceptions of picture fuzziness and spherical fuzziness, as well as T-SF sets, are delineated by way of three functions: the first is membership; the second is abstinence; the third is non-membership. Specifically, the sum of three functions takes a value in the interval [0, 1] in the picture fuzzy model (Cuong, 2014); the square sum of three functions is valued in [0, 1] in the spherical fuzzy model (Gündoğdu and Kahraman, 2019); the summation about three functions to the t-th power (t is a positive integer) is valued in [0, 1] in the T-SF model (Mahmood *et al.*, 2019). The residual term (i.e. the length of the real unit interval excluding the belonging, abstinence, and non-belonging parts) is regarded as the grade of indeterminacy. The T-SF configuration reduces to the picture fuzzy and spherical fuzzy models with the conditions that $t = 1$ and $t = 2$, respectively. In consideration of the null degree of abstinence, T-SF sets are mathematically equivalent to q-rung orthopair fuzzy sets. Taking these discussions into consideration, the concept of T-SF sets is an all-encompassing structure of the before-mentioned non-standard fuzzy models (Chen *et al.*, 2021; Garg *et al.*, 2021; Ullah *et al.*, 2020a, 2020b; Zeng *et al.*, 2020).

From an alternative perspective, the concept of neutrosophic sets, incipiently propounded by Smarandache (2005a, 2005b), is worthy of note as well. The conception of neutrosophic sets can be deemed to be a generalized conformation of unification toward intuitionistic fuzzy logic (Jana *et al.*, 2021; Nabeeh *et al.*, 2021; Pamucar *et al.*, 2020). The primary scheme behind neutrosophic logic is to describe the features of a three-dimensional neutrosophic space (Pamucar *et al.*, 2020; Smarandache, 2019). Specifically, three dimensions of the space characterize the grade of truth-membership, the grade of falsehood-membership, and the grade of indeterminacy-membership that are equivalent to positive, negative, and refusal memberships, respectively, under uncertainty (Chen, 2021; Qin and Wang, 2020). In particular, these grades of membership are independent of each other in essence (Karaaslan and Hunu, 2020; Şahin and Liu, 2017). Notably, the most obvious differentiation between intuitionistic fuzzy logic and neutrosophic logic lies in whether the three membership degrees are independent or dependent (Karaaslan and

Hunu, 2020; Pamucar *et al.*, 2020; Smarandache, 2019). In view of the dependent components in intuitionistic fuzzy logic, when one membership degree changes, the other membership degrees need to be changed accordingly to meet the restriction for keeping their total sum being up to 1 (Atanassov, 1986; Smarandache, 2019). On the contrary, making allowance for the independent components in neutrosophic logic, when one membership degree becomes different, the other membership degrees are unnecessary to change accordingly, wherein the summation of three membership degrees is always up to 3 (Smarandache, 2005a, 2005b, 2019). When the decision maker evaluates the choice options, there is bound to be a certain degree of relevance in the assessments of the advantages and disadvantages of the options with respect to specific judgment criteria. In other words, it is impossible for the favourable and unfavourable evaluations of the same subject matter to be unrelated. Therefore, in neutrosophic logic, the assumption that the grades of positive membership and negative membership meet independence is not appropriate for decision-making problems. More precisely, the mechanism involving independent components is not suitable for practical multiple-criteria choice problems, for the reason that the interactions surrounded by three grades of membership have objective reality included in human appraisals and judgments to a great extent (Chen, 2021). The T-SF framework provides an all-encompassing model including intuitionistic fuzzy sets along with certain non-standard fuzzy sets; thus, it would be more appropriate than the neutrosophic framework to portray assessment information for multiple-criteria choice analysis in uncertain circumstances.

Due to the comprehensiveness of T-SF sets, the T-SF configuration serves an important tool to manipulate convoluted uncertainties for formulating and solving multiple-criteria choice problems. By way of illustration, Ali *et al.* (2020) put forward aggregation operators in complex T-SF settings for the sake of managing multiple-criteria evaluation affairs. With the assistance of the generalized parameter contained in T-SF sets, Chen *et al.* (2021) carried forward certain useful geometric aggregation operators with the aim of multiple-criteria choice analysis. Garg *et al.* (2021) launched several beneficial T-SF power aggregation operators to make headway for multiple-criteria evaluation and appraisement. Guleria and Bajaj (2021) progressed aggregation operators for T-SF soft sets to tackle decision-making issues. Ju *et al.* (2021) gave impetus to a T-SF TODIM (i.e. interactive and multiple-criteria decision making in Portuguese) technique for facilitating group decision-making tasks under incomplete preference information. Khan *et al.* (2021b) set forth a fresh evaluation approach using the agency of T-SF Schweizer-Sklar power Heronian aggregation operators for uncertain decisions. Liu *et al.* (2021a) promoted new and creative Muirhead mean aggregation operations via a complex 2-tuple linguistic structure in T-SF circumstances to treat decision analysis issues. Liu *et al.* (2021b) brought forward Maclaurin symmetric aggregation operators with normal T-SF numbers in the support of uncertain decision making. Munir *et al.* (2020) evolved T-SF Einstein hybrid aggregating operations to support the determination of choice options. Munir *et al.* (2021) propounded a T-SF decision-aiding algorithm on grounds of interactive geometric aggregation operations along with associated immediate probabilities. Özlü and Karaaslan (2021) delivered correlation coefficients concerning T-SF type-2 hesitant information and exploited these notions for solving an issue involving clustering the choice options. Wang and Chen (2021) propounded a T-SF ELECTRE (i.e. ELimination Et Choice Translating REality) outranking model for decision analysis with multiple criteria. On account of the advancement of T-SF decision models and techniques, this paper intends to develop an innovative multiple-criteria choice analysis approach using the agency of the T-SF theory for determining the predominance ranks of choice options or alternatives.

1.2. *Brief Review of the REGIME Methodology*

The REGIME method, incipiently propounded by Hinloopen *et al.* (1983a, 1983b), is a well-established technique concerning multiple-criteria choice analysis, especially for qualitative information (Alinezhad and Khalili, 2019; Oztaysi *et al.*, 2021; Tsigdinos and Vlastos, 2021). Based on an efficient and convenient-to-use approach of paired comparisons for choice options, the REGIME method manipulates qualitative information (such as ordinal data) in a mathematically reasonable way (Oztaysi *et al.*, 2022). Of course, the REGIME method accepts both qualitative and quantitative data, and the implementation procedure of this method is simple and easy to understand (Esangbedo *et al.*, 2021; Kamran *et al.*, 2017; Oztaysi *et al.*, 2021). The characteristic of the classical REGIME framework is the formation of a REGIME matrix that collects outcomes about paired comparisons of choice options in an impact matrix (Esangbedo *et al.*, 2021). Herein, the impact matrix elucidates the effect measurements of choice options on an amalgamation of quantitative and qualitative judgment criteria manifested in ordinal values (Alinezhad and Khalili, 2019; Tsigdinos and Vlastos, 2021). REGIME can conclusively generate a complete list of ranking for choice options via comparing the pairs with selected judgment criteria (Esangbedo *et al.*, 2021; Kamran *et al.*, 2017).

There was something unique about the theory of the REGIME methodology, which constitutes a distinctive outranking-based model for multiple-criteria evaluation and choice analysis. As is well known, the ELECTRE and the PROMETHEE (i.e. Preference Ranking Organization METHod for Enrichment of Evaluations) are widely employed outranking-based models. Compared with these two outranking-based models, the REGIME method exploits a mixed approach with combining the logit analysis and Kendall's paired comparisons based on ordinal data (Asgharizadeh *et al.*, 2014; Aspen *et al.*, 2015; Hinloopen *et al.*, 1983a). In this regard, a favourable feature possessed by the REGIME is its ability to utilize hybrid qualitative and quantitative data without requiring to convert the qualitative information into quantitative values (Esangbedo *et al.*, 2021; Kamran *et al.*, 2017; Oztaysi *et al.*, 2022). Moreover, the REGIME is capable of conducting an adaptive analysis (Kamran *et al.*, 2017) because it can render a complete predominance ranking for choice options or alternatives supported by paired comparisons with selected criteria in miscellaneous decision scenarios (Briamonte *et al.*, 2021; Wątróbski *et al.*, 2019). Especially, utilizing the REGIME method can generate undisputed consequences, so the dominant choice will be identified for the most cases (Hinloopen and Nijkamp, 1990; Wątróbski *et al.*, 2019). Over and above that, the REGIME method has been proved as an efficacious technique to resolve various evaluation and decision-making affairs (Frank, 2014; Oztaysi *et al.*, 2021; Tsigdinos and Vlastos, 2021). For these reasons, this research makes an effort at choosing the REGIME as the basic framework for extending to T-SF decision environments and conducting specialized decision-making tasks.

The REGIME has been smoothly exploited in the treatment of multiple-criteria evaluation problems (Hinloopen and Nijkamp, 1990), such as assessment and prioritization of coastal areas (Hinloopen *et al.*, 1983b), regional sustainable resource policy (Akgün *et al.*, 2012), assessment of alternative wind park locations (Stratigea and Grammatikogiannis, 2012), environmental management on wastewater from agriculture (Massei *et al.*, 2014), economic-ecological sustainability for rural development (Akgün *et al.*, 2015), sawability related to ornamental and building stones (Kamran *et al.*, 2017), and strategic road network of a metropolitan region (Tsigdinos and Vlastos, 2021). On the flip-side, the REGIME framework has been generalized to fuzzy environments, such as the directly extended REGIME methods involving Pythagorean fuzziness (Oztaysi *et al.*, 2021) and spherical fuzziness (Oztaysi *et al.*, 2022). These fuzzy extensions of the REGIME method would form a basis for further advancement under T-SF uncertainties. Even though the usefulness and effectiveness of the REGIME technique have been demonstrated in the aforesaid literature, the advancement of the REGIME methodology has not been investigated yet in T-SF decision contexts. In view of this, it is critically important in the establishment of a T-SF REGIME approach as the increasing complexity and high-order fuzziness in realistic decision-making processes.

1.3. *Research Gaps and Motivations*

In terms of fuzzy community, the theory of T-spherical fuzziness contains a very comprehensive account of several non-standard fuzzy configurations. The advancements in the study of the fuzzy decision-making field are crucial for the subject of multiple-criteria choice analysis because of the necessity of managing uncertain information in real decisions. According to the investigations regarding the aforementioned literature, the research gaps and motivations for this paper are threefold:

1. With the increasing usage of the T-SF theory in decision analysis, developing an appropriate multiple-criteria choice technique for resolving preference predominance rankings has become more critical under remarkably complicated uncertainties. This generates the first motivation in order to span the gap in such research topics. Namely, there would be generally high demand for exploiting the T-SF theory in multiple-criteria choice analysis, which has been echoed by numerous decision-making models and methods in T-SF circumstances.
2. As uncertainties in real decisions are common, the classical REGIME method would be criticized for its ignorance of ambiguous and equivocal information and lack of proper manipulation. On account of this technical gap, as technology requirements in realistic uncertain contexts increases, so does demand for more precise modelling of uncertainties and efficient manipulation of subjective assessments, which brings about the second motivation.
3. There are now a mushrooming number of studies that shed some light on the subject of decision models predicated on T-SF sets; however, little research has been done

on adapting the classic REGIME methodology to a T-SF uncertain context. The existing multiple criteria evaluation and choice methods involving T-spherical fuzziness displayed an utter lack of interest in the extension of the REGIME technique to T-SF environments, which gives rise to the third motivation.

From this basis, this paper attempts to making the REGIME accommodate to the realities of intricate uncertain circumstances. Moreover, this paper contrives of a novel T-SF REGIME method using several beneficial notions for conducting multiple-criteria choice analysis under T-SF uncertainty.

1.4. *Originality and Contributions*

The primary purpose of this study is to specify a suitable measurement system for complex T-SF information and launch an innovative T-SF REGIME method for addressing multiple criteria evaluation and choice issues. It is worth mentioning that this paper intends to exploit the main structure of REGIME to adapt to T-SF decision environments. Because this paper desires to manipulate complex T-SF uncertain information, the evolved REGIME method should have differing adaptive notions corresponding to the differing phases of the core REGIME procedure. First, this paper constitutes a T-SF multiple-criteria choice problem involving judgment criteria and choice options, along with relevant T-SF evaluation values embedded in each T-SF characteristic. Next, in order to differentiate such T-SF assessment information, this study exploits a beneficial score function that signifies grades of satisfaction, neutral satisfaction, dissatisfaction, and refusal membership for utilizing T-SF uncertain information adequately. In accordance with score functions and accuracy values, this study identifies the superiority criteria supported by paired predominance relationships. Soon afterward, this study amalgamates the discrepancy between score functions with the total weights of superiority criteria for enriching the notion of superiority indices. Then, this paper originates an efficacious superiority identifier capable of measuring the relative attractiveness between T-SF characteristics. And following the superiority identifier, this paper presents the notions of REGIME identifiers and REGIME vectors to lay the foundations of a REGIME matrix and establish a guide index capable of measuring the relative fittingness for T-SF characteristics. This study conceives two beneficial procedures for the T-SF REGIME I and II prioritization to yield the eventual partial- and complete-preference rankings, respectively, for available options. By way of the Boolean matrices based on superiority identifiers and guide indices, this paper confirms valuable outranking relationships for generating the T-SF REGIME I predominance ranks for choice options. Moreover, this paper evolves the T-SF REGIME II predominance ranks of options on the basis of the net superiority identifier and the net guide index. To demonstrate the conceivability and advantages of the evolved T-SF REGIME methodology in pragmatic decisions, this paper explores a realistic problem concerning the company selection for plant-building. The effectiveness and constructiveness of the developed techniques can be illustrated through the agency of a comparative analysis.

1.5. *Structure of This Research*

The remainder of this research paper is exhibited systematically along these lines. Section 2 provides an introductory description of some rudimental notions about T-SF sets. Section 3 puts forward a novel T-SF REGIME method through the utility of the score function-based superiority identifiers and guide indices. Section 4 executes the initiated technique to handle the selection problem of companies for setting up food processing plants and then carry out a comparative analysis with the T-SF versions of other decision-making methods. Section 5 concludes this research work with certain concluding remarks, academic contributions, limitations, and future research directions.

2. Notion of T-SF Sets: Preliminaries

T-SF sets contribute a magnificent model to comprehensively manage the equivocation and indefiniteness accompanied by multiple criteria evaluation and assessment activities. Herein, this section intends to describe preliminary notations related to T-SF sets. Throughout this work, the notations μ, η, ν, and γ represent grades of positive membership, neutral membership (i.e. abstinence), negative membership, and refusal membership, respectively, in the unit interval [0, 1]. Relevant concepts can give assistance to construct the initiated T-SF REGIME method.

Definition 1 (Cuong, 2014). Let a finite nonempty set U represent the domain of discourse containing an element u. A picture fuzzy set P in U is stated precisely like this:

$$P = \left\{\left\langle u, \left(\mu_P(u), \eta_P(u), \nu_P(u)\right)\right\rangle \,\middle|\, 0 \leqslant \mu_P(u) + \eta_P(u) + \nu_P(u) \leqslant 1, u \in U\right\}, \quad (1)$$

where $\gamma_P(u) = 1 - \mu_P(u) - \eta_P(u) - \nu_P(u)$. The triplet $(\mu_P(u), \eta_P(u), \nu_P(u))$ is stipulated as a picture fuzzy number.

Definition 2 (Gündoğdu and Kahraman, 2019). A spherical fuzzy set S in U is listed below:

$$S = \left\{\left\langle u, \left(\mu_S(u), \eta_S(u), \nu_S(u)\right)\right\rangle \,\middle|\, 0 \leqslant \left(\mu_S(u)\right)^2 + \left(\eta_S(u)\right)^2 + \left(\nu_S(u)\right)^2 \leqslant 1, u \in U\right\}, \quad (2)$$

where $\gamma_T(u) = \sqrt{1 - (\mu_S(u))^2 - (\eta_S(u))^2 - (\nu_S(u))^2}$. The triplet $(\mu_S(u), \eta_S(u), \nu_S(u))$ is explicated as a spherical fuzzy number.

Definition 3 (Mahmood *et al.*, 2019). A T-SF set T in U is delineated as shown:

$$T = \left\{\left\langle u, \left(\mu_T(u), \eta_T(u), \nu_T(u)\right)\right\rangle \,\middle|\, 0 \leqslant \left(\mu_T(u)\right)^z + \left(\eta_T(u)\right)^z + \left(\nu_T(u)\right)^z \leqslant 1, u \in U\right\}, \quad (3)$$

where $\gamma_T(u) = \sqrt[z]{1 - (\mu_T(u))^z - (\eta_T(u))^z - (\nu_T(u))^z}$, and $z \in Z^+$ (Z^+: a collection of positive integers). The triplet $(\mu_T(u), \eta_T(u), \nu_T(u))$ is named a T-SF number, denoted as $t(u)$.

If $z = 1$, the T-SF set T in Eq. (3) reduces to the picture fuzzy set P in Eq. (1). If $z = 2$, the T-SF set T in Eq. (3) reduces to the spherical fuzzy set S in Eq. (2). From this basis, picture fuzzy sets and spherical fuzzy sets are referred to as the special cases of the concepts of T-SF sets. On the flip-side, if $\eta_T(u) = 0$, the T-SF set T condenses into a q-rung orthopair fuzzy version. And by the same token, making allowance for the precondition that $\eta_T(u) = 0$, the T-SF set T reduces to intuitionistic, Pythagorean, and Fermatean fuzzy sets if $z = 1, 2, 3$, respectively. Therefore, the theory of T-spherical fuzziness can be referred to as a generalized configuration with respect to the well-received non-standard fuzzy sets.

Definition 4 (Mahmood *et al.*, 2019). The score value $Sv(t(u))$ and the accuracy value $Av(t(u))$ of a T-SF number $t(u)$ are portrayed precisely in this manner:

$$Sv\big(t(u)\big) = \big(\mu_T(u)\big)^z - \big(\nu_T(u)\big)^z, \tag{4}$$

$$Av\big(t(u)\big) = \big(\mu_T(u)\big)^z + \big(\eta_T(u)\big)^z + \big(\nu_T(u)\big)^z, \tag{5}$$

where $Sv(t(u)) \in [-1, 1]$ and $Av(t(u)) \in [0, 1]$.

The notions of $Sv(t(u))$ and $Av(t(u))$ can facilitate the establishment of a comparison rule of T-SF numbers. The greater the score value $Sv(t(u))$ is, the higher the T-SF number $t(u)$ will be. In the same fashion, the greater the accuracy value $Av(t(u))$ is, the higher the $t(u)$ will be. By way of explanation, let $t(u_1)$ and $t(u_2)$ denote two T-SF numbers. The following rules can be employed to compare $t(u_1)$ and $t(u_2)$:

1. If $Sv(t(u_1)) > Sv(t(u_2))$, then $t(u_1)$ is higher than $t(u_2)$;
2. If $Sv(t(u_1)) = Sv(t(u_2))$, then:
   a) If $Av(t(u_1)) > Av(t(u_2))$, then $t(u_1)$ is higher than $t(u_2)$;
   b) If $Av(t(u_1)) = Av(t(u_2))$, then $t(u_1)$ is equal to $t(u_2)$.

However, the grades of neutral membership $\eta_T(u)$ and refusal membership $\gamma_T(u)$ are not involved in the specification of $Sv(t(u))$. Furthermore, the grade of refusal membership $\gamma_T(u)$ is not contained in the specification of $Av(t(u))$. On the grounds of this, the aforesaid comparison rule would lose some decision information contained in T-SF numbers. In an attempt to avoid loss of influential information, Zeng *et al.* (2019) initiated a new formulation of T-SF score functions and stated several useful properties for the sake of comparing T-SF numbers. Zeng *et al.* (2019) took advantage of a curve function with the format $Cf(x) = e^x/(e^x + 1)$ whose range is always in the open interval $(0, 1)$. Moreover, it is recognized that $Cf(x) + Cf(-x) = 0$, which follows directly that $Cf(0) = 0.5$. In particular, the curve function $Cf(x)$ is rigorously upswing in domain of real numbers. Supported by the curve function $Cf(x)$, Zeng *et al.* (2019) propounded an efficacious score function $Sf(t(u))$ of a T-SF number $t(u)$ in U.

Definition 5 (Zeng *et al.*, 2019). The score function $Sf(t(u))$ is delineated as shown:

$$Sf\big(t(u)\big) = \big(\mu_T(u)\big)^z - \big(\eta_T(u)\big)^z - \big(\nu_T(u)\big)^z + \left(\frac{e^{(\mu_T(u))^z-(\eta_T(u))^z-(\nu_T(u))^z}}{e^{(\mu_T(u))^z-(\eta_T(u))^z-(\nu_T(u))^z}+1} - \frac{1}{2}\right)\big(\gamma_T(u)\big)^z, \tag{6}$$

where $Sf(t(u)) \in [-1, 1]$. Moreover, $Sf(t(u))$ reduces a score value for picture fuzzy and spherical fuzzy contexts if $z = 1, 2$, respectively. When $\eta_T(u) = 0$, $Sf(t(u))$ reduces a score value for intuitionistic, Pythagorean, and Fermatean fuzzy contexts if $z = 1, 2, 3$, respectively.

3. Proposed T-SF REGIME Methodology

This section makes an effort to develop a novel T-SF REGIME method for managing multiple-criteria choice issues under intricate uncertain circumstances. This section designs some beneficial concepts to plan an approach for determining predominance relationships among the T-SF evaluation values of given alternatives. An efficacious T-SF REGIME procedure is evolved to help decision makers arrive at a choice.

The proposed T-SF REGIME method consists of five phases: (i) organization of a multiple-criteria choice problem, (ii) establishment of score function-based superiority criteria, (iii) determination of the superiority measurements, (iv) identification of the REGIME matrix and guide indices, and (v) construction of the eventual partial/complete ranking. The research focuses in the five phases are depicted in the T-SF REGIME framework of the evolved methodology. The analytical framework of the advanced T-SF REGIME is portrayed in Fig. 1. In the first phase, this paper constitutes a multiple-criteria choice issue under complicated T-SF uncertainty. In the second phase, this paper utilizes a beneficial score function to differentiate T-SF information for pinpointing the superiority criteria. In the third phase, the developed approach aims to exploit the score function-based collection of superiority criteria to delineate the notions of a superiority index and a superiority identifier. In the fourth phase, this paper utilizes the sign of the contrast between score functions to form a REGIME matrix and then generate a guide index. In the final phase, this study produces the T-SF REGIME I and II prioritization mechanisms in the expectation of yielding partial-preference and complete-preference predominance rankings, respectively, for choice options. To draw as a logical conclusion about the T-SF REGIME I eventual predominance ranks, this study launches the conception of a superiority-based Boolean matrix and a guide-based Boolean matrix. This paper unfolds the notions of net superiority identifiers and net guide indices for deducing the T-SF REGIME II ranking among choice options. At the beginning, a multiple-criteria choice task would be constituted in T-SF decision circumstances.

Consider the mathematical description of a multiple-criteria choice analysis task in the first phase. Let $A = \{a_1, a_2, \ldots, a_m\}$ expound a collection of m ($\geqslant$2) choice options or alternatives. Moreover, let $C = \{c_1, c_2, \ldots, c_n\}$ represent a collection of n ($\geqslant$2) judgment criteria that elucidate factors and characteristics for evaluating the choice options

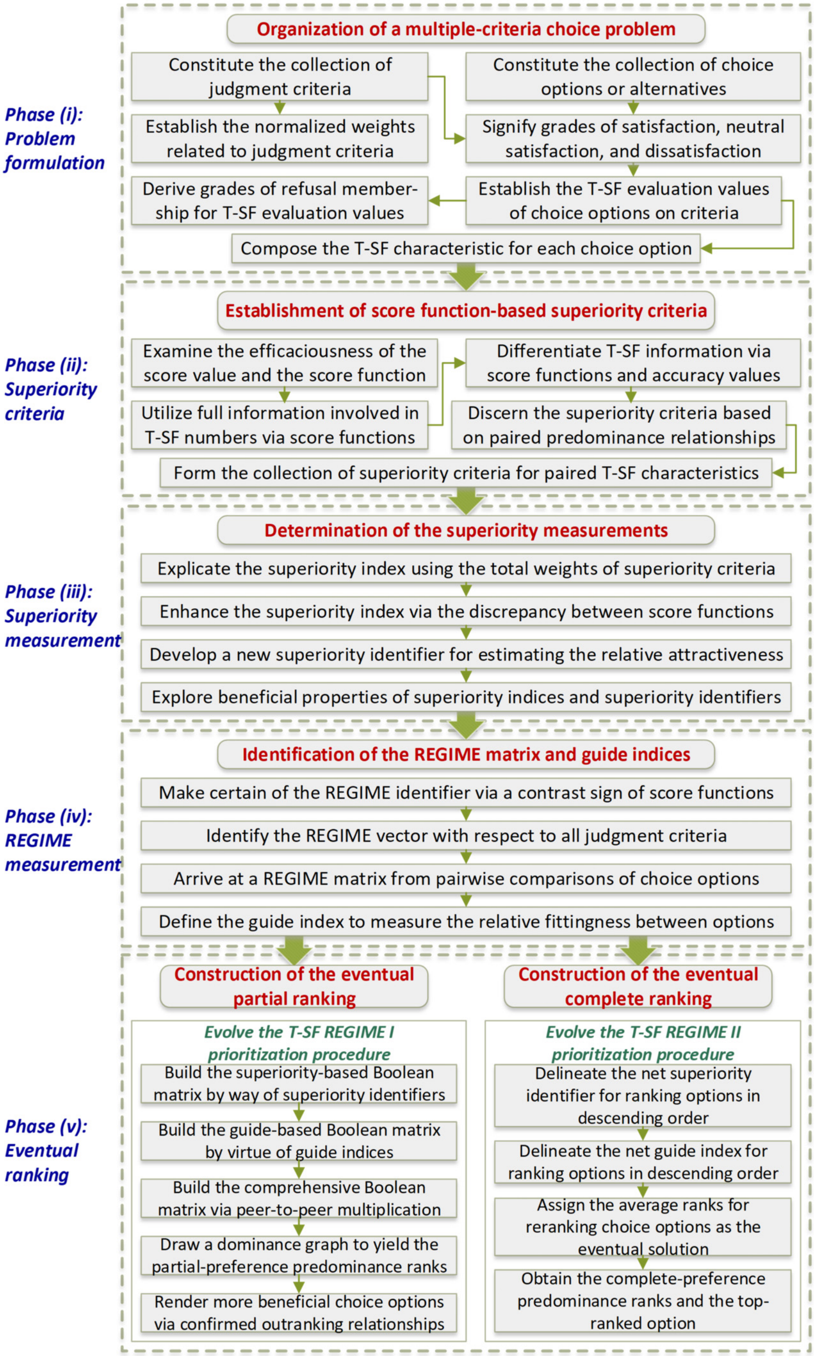


Fig. 1. The framework of the evolved T-SF REGIME methodology.

from which decision makers are choosing. The weight $w_j \in [0, 1]$ associated with each judgment criterion $c_j \in C$ fulfills the normalized condition $\sum_{j=1}^{n} w_j = 1$. In a fundamental manner, the measures concerning performance ratings of choice options can be determined using a satisfaction survey through a questionnaire designed to understand what the decision maker thinks about these alternatives in terms of each criterion. In the uncertain contexts with T-spherical fuzziness, the T-SF evaluation value t_{ij} of a choice option $a_i \in A$ in conjunction with a judgment criterion $c_j \in C$ is precisely stated via a T-SF number $(\mu_{ij}, \eta_{ij}, \nu_{ij})$ that is constituted by the grade of satisfaction μ_{ij}, grade of neutral satisfaction η_{ij}, and grade of dissatisfaction ν_{ij}. Moreover, the corresponding grade of refusal membership is given by $\gamma_{ij} = \sqrt[z]{1 - (\mu_{ij})^z - (\eta_{ij})^z - (\nu_{ij})^z}$. The T-SF characteristic T_i of a_i for $z \in Z^+$ is expressed in this manner:

$$T_i = \left\{\left\langle c_j, (\mu_{ij}, \eta_{ij}, \nu_{ij})\right\rangle \,\middle|\, 0 \leqslant (\mu_{ij})^z + (\eta_{ij})^z + (\nu_{ij})^z \leqslant 1, \forall c_j \in C\right\}. \tag{7}$$

The REGIME mechanism exploits a comparison of any two choice options in pairs. Next, an eventual predominance ranking of all available options is co-ascertained through the mutual comparisons of alternatives. With the purpose of building a new mechanism of the proposed T-SF REGIME method, this paper would first launch an identification approach of a regime. There is usually no obvious single dominant choice option in practical decision-making affairs. Therefore, one needs a better way to proceed with T-SF evaluation values in pairwise comparisons that focus on differences between choice options with respect to the judgment criteria. The score function exhibited by Zeng *et al.* (2019) can fully utilize the information about grades of positive, neutral, negative, and refusal memberships that quantify belongingness of a judgment criterion in the domain of discourse to a T-SF characteristic. Let us examine the effectiveness of the score function $Sf(t_{ij})$ in comparison with the score value $Sv(t_{ij})$ through the agency of the subsequent example.

Example 1. Consider that two T-SF evaluation values $t_{ij} = (0.6, 0.4, 0.5)$ and $t_{i'j} = (0.6, 0.1, 0.5)$ with $z = 3$. By virtue of Eq. (4), it was received that $Sv(t_{ij}) = 0.6^3 - 0.5^3 = 0.0910$ and $Sv(t_{i'j}) = 0.6^3 - 0.5^3 = 0.0910$. Next, the grades of refusal membership corresponding to t_{ij} and $t_{i'j}$ were separately computed as follows: $\gamma_{ij} = \sqrt[3]{1 - 0.6^3 - 0.4^3 - 0.5^3} = 0.8411$ and $\gamma_{i'j} = \sqrt[3]{1 - 0.6^3 - 0.1^3 - 0.5^3} = 0.8698$. With the aid of Eq. (6), it was obtained that:

$$Sf(t_{ij}) = 0.6^3 - 0.4^3 - 0.5^3 + \left(\frac{e^{0.6^3-0.4^3-0.5^3}}{e^{0.6^3-0.4^3-0.5^3}+1} - \frac{1}{2}\right)0.8411^3 = 0.4272,$$

$$Sf(t_{i'j}) = 0.6^3 - 0.1^3 - 0.5^3 + \left(\frac{e^{0.6^3-0.1^3-0.5^3}}{e^{0.6^3-0.1^3-0.5^3}+1} - \frac{1}{2}\right)0.8698^3 = 0.5416.$$

Obviously, due to $Sv(t_{ij}) = Sv(t_{i'j})$, failure to discriminate between the superiority degrees of t_{ij} and $t_{i'j}$ leads to the decreased usefulness of the score value as stated by Definition 4. In contrast, the employment of the score function as attested by Definition 5 was able to differentiate between the superiority degrees of t_{ij} and $t_{i'j}$ because

$Sf(t_{ij}) < Sf(t_{i'j})$. This example has demonstrated the effectuality of the score function due to the fact that the specification of $Sf(t_{ij})$ can utilize full information contained in T-SF evaluation values.

The research focus in the second phase is the differentiation of T-SF evaluation values for discerning the superiority criteria. For the most part, one can determine a predominance relationship portrayed by the rationale of "the higher, the better". A T-SF evaluation value is considered to be superior to another T-SF evaluation value in case that the corresponding score function is more significant. When two T-SF evaluation values have an identical score function value, the corresponding accuracy values would be exploited to examine the superiority of these T-SF evaluations. That is, a T-SF evaluation value is regarded to be more significant to another T-SF evaluation value if its accuracy value is larger. Considering two T-SF evaluation values of choice options $a_i, a_{i'} \in A$ in conjunction with a judgment criterion $c_j \in C$, the comparison rule concerned with t_{ij} and $t_{i'j}$ is elucidated as follows:

1. If $Sf(t_{ij}) > Sf(t_{i'j})$, then t_{ij} is superior to $t_{i'j}$;
2. If $Sf(t_{ij}) = Sf(t_{i'j})$, then:
   a) If $Av(t_{ij}) > Av(t_{i'j})$, then t_{ij} is superior to $t_{i'j}$;
   b) If $Av(t_{ij}) = Av(t_{i'j})$, then the superiority of t_{ij} and $t_{i'j}$ becomes equal.

Definition 6. Consider two T-SF characteristics T_i and $T_{i'}$ of the choice options $a_i, a_{i'} \in A$, where $T_i = \{\langle c_j, t_{ij}\rangle \mid c_j \in C\}$ and $T_{i'} = \{\langle c_j, t_{i'j}\rangle \mid c_j \in C\}$. Let $Cs(T_i, T_{i'})$ denote a collection of superiority criteria in which T_i is at least as good as $T_{i'}$; it is identified along these lines:

$$Cs(T_i, T_{i'}) \\ = \left\{c_j \,\middle|\, Sf(t_{ij}) > Sf(t_{i'j}) \text{ or } \big(Sf(t_{ij}) = Sf(t_{i'j}) \text{ and } Av(t_{ij}) \geqslant Av(t_{i'j})\big)\right\}. \quad (8)$$

Theorem 1. *The complement of $Cs(T_i, T_{i'})$, which is named a collection of inferiority criteria, contains all judgment criteria for which T_i performs worse than $T_{i'}$. The collection of inferiority criteria $Ci(T_i, T_{i'})$ is written like this*:

$$Ci(T_i, T_{i'}) \\ = \left\{c_j \,\middle|\, Sf(t_{ij}) < Sf(t_{i'j}) \textit{ or } \big(Sf(t_{ij}) = Sf(t_{i'j}) \textit{ and } Av(t_{ij}) < Av(t_{i'j})\big)\right\}. \quad (9)$$

Without loss of generality, assume that the equalities $Sf(t_{ij}) = Sf(t_{i'j})$ and $Av(t_{ij}) = Av(t_{i'j})$ will not happen at the same time for each $c_j \in C$. Then, the collection of superiority criteria $Cs(T_i, T_{i'})$ satisfies the following properties:

(T1.1) $Cs(T_i, T_{i'}) \cap Cs(T_{i'}, T_i) = \emptyset$;
(T1.2) $Cs(T_i, T_{i'}) \cup Cs(T_{i'}, T_i) = C$;
(T1.3) $Cs(T_{i'}, T_i) = C \backslash Cs(T_i, T_{i'})$ (*i.e. the set difference of C and $Cs(T_i, T_{i'})$*);
(T1.4) $Cs(T_i, T_{i'}) = Ci(T_{i'}, T_i)$.

Proof. Because two equalities $Sf(t_{ij}) = Sf(t_{i'j})$ and $Av(t_{ij}) = Av(t_{i'j})$ do not happen simultaneously for a specific criterion c_j, it can be received that:

$$\begin{aligned}
&Cs(T_{i'}, T_i)\\
&\quad = \{c_j \mid Sf(t_{i'j}) > Sf(t_{ij}) \text{ or } (Sf(t_{i'j}) = Sf(t_{ij}) \text{ and } Av(t_{i'j}) > Av(t_{ij}))\}\\
&\quad = \{c_j \mid Sf(t_{ij}) < Sf(t_{i'j}) \text{ or } (Sf(t_{ij}) = Sf(t_{i'j}) \text{ and } Av(t_{ij}) < Av(t_{i'j}))\}.
\end{aligned}$$

Clearly, it is deduced that $Cs(T_i, T_{i'}) \cap Cs(T_{i'}, T_i) = \emptyset$ and $Cs(T_i, T_{i'}) \cup Cs(T_{i'}, T_i) = C$; moreover, the latter leads to the relative complement of $Cs(T_i, T_{i'})$ in conjunction with the set C, written $C \backslash Cs(T_i, T_{i'})$. The set of elements in C that are not in $Cs(T_i, T_{i'})$ is derived by $Cs(T_{i'}, T_i) = C \backslash Cs(T_i, T_{i'})$. Thus, (T1.1)–(T1.3) are valid. (T1.4) can be easily concluded because:

$$\begin{aligned}
&Cs(T_i, T_{i'})\\
&\quad = \{c_j \mid Sf(t_{ij}) > Sf(t_{i'j}) \text{ or } (Sf(t_{ij}) = Sf(t_{i'j}) \text{ and } Av(t_{ij}) > Av(t_{i'j}))\}\\
&\quad = \{c_j \mid Sf(t_{i'j}) < Sf(t_{ij}) \text{ or } (Sf(t_{i'j}) = Sf(t_{ij}) \text{ and } Av(t_{i'j}) < Av(t_{ij}))\}\\
&\quad = Ci(T_{i'}, T_i).
\end{aligned}$$

Thus, the theorem is proved. □

EXAMPLE 2. Consider a collection of judgment criteria $C = \{c_1, c_2, c_3, c_4\}$ and two T-SF characteristics T_i and $T_{i'}$ of the choice options $a_i, a_{i'} \in A$. Herein, T_i and $T_{i'}$ are given by:

$$\begin{aligned}
T_i = \{&\langle c_1, (0.5, 0.3, 0.4)\rangle, \langle c_2, (0.7, 0.5, 0.2)\rangle, \langle c_3, (0.2, 0.2, 0.8)\rangle,\\
&\langle c_4, (0.8, 0.5, 0.5)\rangle\},\\
T_{i'} = \{&\langle c_1, (0.2, 0.4, 0.7)\rangle, \langle c_2, (0.9, 0.2, 0.5)\rangle, \langle c_3, (0.5, 0.3, 0.7)\rangle,\\
&\langle c_4, (0.3, 0.2, 0.6)\rangle\}.
\end{aligned}$$

In accordance with Eq. (6), the following score functions were acquired: $Sf(t_{i1}) = 0.5626$, $Sf(t_{i2}) = 0.5829$, $Sf(t_{i3}) = -0.2547$, $Sf(t_{i4}) = 0.4339$, $Sf(t_{i'1}) = -0.0638$, $Sf(t_{i'2}) = 0.7042$, $Sf(t_{i'3}) = 0.0632$, and $Sf(t_{i'4}) = 0.2685$. It was acquired that $Cs(T_i, T_{i'}) = \{c_1, c_4\}$ and $Ci(T_i, T_{i'}) = \{c_2, c_3\}$, using Eqs. (8) and (9), respectively. Based on (T1.3), $Cs(T_{i'}, T_i) = C \backslash Cs(T_i, T_{i'}) = \{c_2, c_3\}$. Finally, it was gained that $Ci(T_{i'}, T_i) = Cs(T_i, T_{i'}) = \{c_1, c_4\}$ by way of (T1.4).

Consider the third phase of the superiority measurements in the evolved T-SF REGIME methodology. A significant concept in the classical REGIME mechanism would be the superiority index. This index manifests an extent to which a choice option a_i is superior or equal to $a_{i'}$ merely as the relevant judgment criteria is involved in the collection of superiority criteria $Cs(T_i, T_{i'})$. In the framework of REGIME, the superiority index is explicated using the sum of weights contingent upon the superiority criteria contained in $Cs(T_i, T_{i'})$. This index can simply estimate the relative attractiveness of a_i over

$a_{i'}$ in a rather unsophisticated manner. Nonetheless, this treatment of superiority measurements may ignore the essential nature of T-SF data. The extent of the discrepancy between $Sf(t_{ij})$ and $Sf(t_{i'j})$ should be taken under advisement. That is, a more efficacious approach to proceeding with T-SF information in the proposed REGIME mechanism is to focus on differences between $Sf(t_{ij})$ and $Sf(t_{i'j})$ by means of mutual comparisons, more specifically, by adding the products of the difference $Sf(t_{ij}) - Sf(t_{i'j})$ and the weight w_j for $c_j \in Cs(T_i, T_{i'})$. This improved way delivers a new and creative superiority identifier that can be carried out with a focus on arriving at consequences about the relative attractiveness between choice options.

Definition 7. Consider the normalized weight w_j and the T-SF characteristics T_i and $T_{i'}$. In accordance with the score functions $Sf(t_{ij})$ and $Sf(t_{i'j})$ along with the collection of superiority criteria $Cs(T_i, T_{i'})$, the superiority index $Si(T_i, T_{i'})$ and the superiority identifier $SI(T_i, T_{i'})$ in connection with a_i over $a_{i'}$ are elucidated along these lines:

$$Si(T_i, T_{i'}) = \sum_{c_j \in Cs(T_i, T_{i'})} w_j, \tag{10}$$

$$SI(T_i, T_{i'}) = \sum_{c_j \in Cs(T_i, T_{i'})} w_j \big(Sf(t_{ij}) - Sf(t_{i'j})\big). \tag{11}$$

Theorem 2. *The superiority index $Si(T_i, T_{i'})$ and the superiority identifier $SI(T_i, T_{i'})$ fulfill the subsequent conditions*:

(T2.1) $0 \leqslant Si(T_i, T_{i'}) \leqslant 1$;
(T2.2) $0 \leqslant SI(T_i, T_{i'}) \leqslant 2$.

Proof. Based on (T1.2), it can be recognized that $Cs(T_i, T_{i'})$ is a subset of the collection of judgment criteria C; thus, $Cs(T_i, T_{i'}) \subset C$ in mathematics. In line with the normalized condition, it is deduced that $0 \leqslant \sum_{c_j \in Cs(T_i, T_{i'})} w_j \leqslant \sum_{c_j \in C} w_j = 1$, which brings about $0 \leqslant Si(T_i, T_{i'}) \leqslant 1$. Next, based on Definition 5, the ranges of $Sf(t_{ij})$ and $Sf(t_{i'j})$ are bounded in the interval $[-1, 1]$, which indicates explicitly that $0 \leqslant |Sf(t_{ij}) - Sf(t_{i'j})| \leqslant 2$. From Definition 6, it leads us to understand that $Sf(t_{ij}) \geqslant Sf(t_{i'j})$ for each $c_j \in Cs(T_i, T_{i'})$, which follows that $0 \leqslant Sf(t_{ij}) - Sf(t_{i'j}) \leqslant 2$ for $c_j \in Cs(T_i, T_{i'})$. By reason of the normalized condition of w_j, it concludes that $0 \leqslant \sum_{c_j \in Cs(T_i, T_{i'})} w_j (Sf(t_{ij}) - Sf(t_{i'j})) \leqslant 2$. Therefore, (T2.1) and (T2.2) are correct. The theorem is proved. □

Next, consider the fourth phase of the REGIME matrix formation and the determination of guide indices in the current methodology. In particular, the guide index is employed to measure the relative fittingness between choice options. In accordance with the score functions of T-SF evaluation values, this paper builds the REGIME identifier and the REGIME vector for arriving at a REGIME matrix from paired juxtapositions of choice options in conjunction with judgment criteria, as described below.

Definition 8. Consider the T-SF evaluation values t_{ij} and $t_{i'j}$ contained in the T-SF characteristics T_i and $T_{i'}$, respectively. The REGIME identifier $Ri(t_{ij}, t_{i'j})$ is demarcated as a sign of the contrast outcomes through score functions in this manner:

$$Ri(t_{ij}, t_{i'j}) = \begin{cases} 1 & \text{if } Sf(t_{ij}) > Sf(t_{i'j}), \\ 0 & \text{if } Sf(t_{ij}) = Sf(t_{i'j}), \\ -1 & \text{if } Sf(t_{ij}) < Sf(t_{i'j}). \end{cases} \tag{12}$$

The REGIME vector $Rv(T_i, T_{i'})$ is composed of $Ri(t_{ij}, t_{i'j})$ across all $c_j \in C$ like this:

$$Rv(T_i, T_{i'}) = \big(Ri(t_{i1}, t_{i'1}), Ri(t_{i2}, t_{i'2}), \dots, Ri(t_{in}, t_{i'n})\big). \tag{13}$$

The REGIME matrix Rm is established by taking together the REGIME vectors for all ordered couples of choice options $(a_i, a_{i'})$ (for $i \neq i'$):

$$Rm = \begin{bmatrix} Rv(T_1, T_2) \\ \vdots \\ Rv(T_1, T_m) \\ Rv(T_2, T_1) \\ \vdots \\ Rv(T_2, T_m) \\ \vdots \\ Rv(T_m, T_1) \\ \vdots \\ Rv(T_m, T_{m-1}) \end{bmatrix} = \begin{array}{c} \\ (a_1, a_2) \\ \vdots \\ (a_1, a_m) \\ (a_2, a_1) \\ \vdots \\ (a_2, a_m) \\ \vdots \\ (a_m, a_1) \\ \vdots \\ (a_m, a_{m-1}) \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} Ri(t_{11}, t_{21}) & Ri(t_{12}, t_{22}) & \cdots & Ri(t_{1n}, t_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ Ri(t_{11}, t_{m1}) & Ri(t_{12}, t_{m2}) & \cdots & Ri(t_{1n}, t_{mn}) \\ Ri(t_{21}, t_{11}) & Ri(t_{22}, t_{12}) & \cdots & Ri(t_{2n}, t_{1n}) \\ \vdots & \vdots & \ddots & \vdots \\ Ri(t_{21}, t_{m1}) & Ri(t_{22}, t_{m2}) & \cdots & Ri(t_{2n}, t_{mn}) \\ \vdots & \vdots & \ddots & \vdots \\ Ri(t_{m1}, t_{11}) & Ri(t_{m2}, t_{12}) & \cdots & Ri(t_{mn}, t_{1n}) \\ \vdots & \vdots & \ddots & \vdots \\ Ri(t_{m1}, t_{m-1,1}) & Ri(t_{m2}, t_{m-1,2}) & \cdots & Ri(t_{mn}, t_{m-1,n}) \end{bmatrix} \end{array}. \tag{14}$$

Theorem 3. *The REGIME identifier $Ri(t_{ij}, t_{i'j})$ in the REGIME matrix Rm possesses the property of $Ri(t_{ij}, t_{i'j}) + Ri(t_{i'j}, t_{ij}) = 0$ for all ordered couples of $(a_i, a_{i'})$ $(i \neq i')$ in connection with each $c_j \in C$.*

Proof. Based on Eq. (12), it readily draws as a logical conclusion that $Ri(t_{ij}, t_{i'j}) + Ri(t_{i'j}, t_{ij}) = 0$. In particular, one half of the number of arrays in the REGIME matrix Rm can be deduced from the other half by reason of $Ri(t_{i'j}, t_{ij}) = -Ri(t_{ij}, t_{i'j})$. Therefore, the arrays in Rm are not independent from each other. The theorem is proved. □

DEFINITION 9. Considering the normalized weight w_j for each $c_j \in C$, the guide index $GI(T_i, T_{i'})$ is determined through the utility of the REGIME identifier $Ri(t_{ij}, t_{i'j})$ like this:

$$GI(T_i, T_{i'}) = \sum_{j=1}^{n} w_j \cdot Ri(t_{ij}, t_{i'j}). \tag{15}$$

Theorem 4. *The guide index $GI(T_i, T_{i'})$ and its relevant concept meet some properties*:

(T4.1) $-1 \leqslant GI(T_i, T_{i'}) \leqslant 1$;
(T4.2) $GI(T_i, T_{i'}) + GI(T_{i'}, T_i) = 0$;

(T4.3) $\sum_{j=1}^{n} w_j |Ri(t_{ij}, t_{i'j})| = 1$ *if* $Sf(t_{ij}) \neq Sf(t_{i'j})$.

Proof. As per the fact that $\sum_{j=1}^{n} w_j = 1$ and $Ri(t_{ij}, t_{i'j}) \in \{-1, 0, 1\}$, one readily obtains $-1 \leqslant GI(T_i, T_{i'}) \leqslant 1$. Next, based on Theorem 3, it is known that $Ri(t_{ij}, t_{i'j}) + Ri(t_{i'j}, t_{ij}) = 0$, which brings about $GI(T_i, T_{i'}) + GI(T_{i'}, T_i) = \sum_{j=1}^{n} w_j \cdot (Ri(t_{ij}, t_{i'j}) + Ri(t_{i'j}, t_{ij})) = 0$. Herein, (T4.1) and (T4.2) are valid. At last, the presupposition $Sf(t_{ij}) \neq Sf(t_{i'j})$ leads to either $Ri(t_{ij}, t_{i'j}) = 1$ or $Ri(t_{ij}, t_{i'j}) = -1$. In either case, one has $|Ri(t_{ij}, t_{i'j})| = 1$, which indicates that $\sum_{j=1}^{n} w_j |Ri(t_{ij}, t_{i'j})| = 1$, i.e. (T4.3) is correct. The theorem is proved. □

In the final phase, this paper first establishes the T-SF REGIME I prioritization procedure as a means of achieving partial-preference predominance ranks for choice options. Specifically, this paper attempts to constitute two Boolean matrices on the grounds of superiority identifiers and guide indices separately for generating eventual predominance ranks of all available options. First, on the basis of the superiority identifiers $SI(T_i, T_{i'})$ and $SI(T_{i'}, T_i)$, decision makers or analysts may conclude that the choice option a_i is preferred to $a_{i'}$ when $SI(T_i, T_{i'}) \geqslant SI(T_{i'}, T_i)$. By contrast, decision makers or analysts may conclude that a_i is less preferred to $a_{i'}$ when $SI(T_i, T_{i'}) < SI(T_{i'}, T_i)$. Depending on comparison outcomes of the superiority identifier $SI(T_i, T_{i'})$ and its counter version $SI(T_{i'}, T_i)$, this paper delineates the superiority-based Boolean matrix Bs as below.

Definition 10. Given the superiority identifier $SI(T_i, T_{i'})$ for choice options $a_i, a_{i'} \in A$, the superiority-based Boolean matrix Bs involving an entry $Bs(a_i, a_{i'})$ (for $i \neq i'$) would be designated via comparisons of $SI(T_i, T_{i'})$ with $SI(T_{i'}, T_i)$ in this manner:

$$Bs(a_i, a_{i'}) = \begin{cases} 1 & \text{if } SI(T_i, T_{i'}) > SI(T_{i'}, T_i), \\ 0 & \text{if } SI(T_i, T_{i'}) \leqslant SI(T_{i'}, T_i); \end{cases} \tag{16}$$

$$Bs = \begin{bmatrix} - & Bs(a_1, a_2) & \cdots & Bs(a_1, a_m) \\ Bs(a_2, a_1) & - & \cdots & Bs(a_2, a_m) \\ \vdots & \vdots & \ddots & \vdots \\ Bs(a_m, a_1) & Bs(a_m, a_2) & \cdots & - \end{bmatrix}. \tag{17}$$

Theorem 5. *For all $a_i, a_{i'} \in A$, the entry $Bs(a_i, a_{i'})$ in the superiority-based Boolean matrix Bs possesses the following properties*:

(T5.1) $Bs(a_i, a_{i'}) + Bs(a_{i'}, a_i) = 1$ *if* $SI(T_i, T_{i'}) \neq SI(T_{i'}, T_i)$;
(T5.2) $Bs(a_i, a_{i'}) + Bs(a_{i'}, a_i) = 0$ *if* $SI(T_i, T_{i'}) = SI(T_{i'}, T_i)$;
(T5.3) $\sum_{i'=1, i' \neq i}^{m} \sum_{i=1, i \neq i'}^{m} Bs(a_i, a_{i'}) = m(m-1)/2$ *if* $SI(T_i, T_{i'}) \neq SI(T_{i'}, T_i)$.

Proof. When $SI(T_i, T_{i'}) > SI(T_{i'}, T_i)$, it is recognized $Bs(a_i, a_{i'}) = 1$ and $Bs(a_{i'}, a_i) = 0$. On the contrary, $Bs(a_i, a_{i'}) = 0$ and $Bs(a_{i'}, a_i) = 1$ when $SI(T_i, T_{i'}) < SI(T_{i'}, T_i)$. Thus, one obtains $Bs(a_i, a_{i'}) + Bs(a_{i'}, a_i) = 1$. Next, the condition $SI(T_i, T_{i'}) = SI(T_{i'}, T_i)$ gives rise to $Bs(a_i, a_{i'}) = 0$ and $Bs(a_{i'}, a_i) = 0$. Hence, one yields $Bs(a_i, a_{i'}) + Bs(a_{i'}, a_i) = 0$.

From above, (T5.1) and (T5.2) are correct. (T5.3) can be proved in view of the fact that:

$$\sum_{i'=1,i'\neq i}^{m}\sum_{i=1,i\neq i'}^{m} Bs(a_i, a_{i'}) = \sum_{i<i'}^{m}\big(Bs(a_i, a_{i'}) + Bs(a_{i'}, a_i)\big)$$
$$= \frac{(m^2 - m)}{2} = \frac{m(m-1)}{2}.$$

Thus, the theorem is proved. □

Definition 11. Given a guide index $GI(T_i, T_{i'})$ for choice options $a_i, a_{i'} \in A$, the guide-based Boolean matrix Bg involving an entry $Bg(a_i, a_{i'})$ (for $i \neq i'$) would be designated via comparisons of $GI(T_i, T_{i'})$ and $GI(T_{i'}, T_i)$ in this manner:

$$Bg(a_i, a_{i'}) = \begin{cases} 1 & \text{if } GI(T_i, T_{i'}) > GI(T_{i'}, T_i), \\ 0 & \text{if } GI(T_i, T_{i'}) \leqslant GI(T_{i'}, T_i); \end{cases} \tag{18}$$

$$Bg = \begin{bmatrix} - & Bg(a_1, a_2) & \cdots & Bg(a_1, a_m) \\ Bg(a_2, a_1) & - & \cdots & Bg(a_2, a_m) \\ \vdots & \vdots & \ddots & \vdots \\ Bg(a_m, a_1) & Bg(a_m, a_2) & \cdots & - \end{bmatrix}. \tag{19}$$

Theorem 6. *For all $a_i, a_{i'} \in A$, the entry $Bg(a_i, a_{i'})$ in the guide-based Boolean matrix Bg possesses the following properties*:

(T6.1) $Bg(a_i, a_{i'}) + Bg(a_{i'}, a_i) = 1$ *if* $GI(T_i, T_{i'}) \neq GI(T_{i'}, T_i)$;
(T6.2) $Bg(a_i, a_{i'}) + Bg(a_{i'}, a_i) = 0$ (*i.e.* $Bg(a_i, a_{i'}) = Bg(a_{i'}, a_i) = 0$) *if* $GI(T_i, T_{i'}) = GI(T_{i'}, T_i)$;
(T6.3) $\sum_{i'=1,i'\neq i}^{m}\sum_{i=1,i\neq i'}^{m} Bg(a_i, a_{i'}) = m(m-1)/2$ *if* $GI(T_i, T_{i'}) \neq GI(T_{i'}, T_i)$.

Proof. The proving process would be analogous to the proof in Theorem 5. □

By applying a peer-to-peer multiplication operation in conjunction with the entries in Bs and Bg, the comprehensive Boolean matrix Bc is identified along these lines:

$$Bc(a_i, a_{i'}) = Bs(a_i, a_{i'}) \cdot Bg(a_i, a_{i'}), \tag{20}$$

$$Bc = \begin{bmatrix} - & Bc(a_1, a_2) & \cdots & Bc(a_1, a_m) \\ Bc(a_2, a_1) & - & \cdots & Bc(a_2, a_m) \\ \vdots & \vdots & \ddots & \vdots \\ Bc(a_m, a_1) & Bc(a_m, a_2) & \cdots & - \end{bmatrix}. \tag{21}$$

The unit entries in the comprehensive Boolean matrix Bc give a description of the confirmed predominance relationships between choice options. The outcome $Bc(a_i, a_{i'}) = 1$ indicates that a_i is preferred to $a_{i'}$ from both perspectives of superiority identifiers and guide indices. In contrast, the outcome $Bc(a_i, a_{i'}) = 0$ mentions that a_i is indifferent to or less preferred to $a_{i'}$ based on superiority identifiers and guide indices. The partial-preference predominance ranks of choice options can be determined by virtue of the entry $Bc(a_i, a_{i'})$ in Bc, which renders the T-SF REGIME I ranking conclusion.

However, the use of the Boolean matrices Bs, Bg, and Bc would sometimes bring about certain difficulties, because no concrete outcome can be concluded concerning the complete predominance ranking among choice options. The characteristic of a net regime analysis would be the circumvention of such difficulties by determining a net superiority identifier and a net guide index for linear ranking outcomes. Such an approach can draw a definite conclusion about the complete predominance ranking of choice options. This is the subject of the T-SF REGIME II prioritization procedure.

The proposed T-SF REGIME II approach delineates the net superiority identifier $N_{SI}(a_i)$ which measures the degree to which the relative attractiveness of a choice option a_i over the other competing options exceeds the relative attractiveness of the other competing options over a_i. Concurrently, the current T-SF REGIME II approach elucidates the net guide index $N_{GI}(a_i)$ which measures the degree to which the relative fittingness of a_i over the other competing options exceeds the relative fittingness of the other competing options over a_i. These net measurements are capable of generating a complete-preference predominance ranking among choice options for decision aiding in intricate T-SF contexts. To be specific, the net superiority identifier $N_{GI}(a_i)$ of a_i with relevance to the other choice options is mathematically denoted as follows:

$$N_{SI}(a_i) = \sum_{i'=1, i' \neq i}^{m} SI(T_i, T_{i'}) - \sum_{i'=1, i' \neq i}^{m} SI(T_{i'}, T_i). \tag{22}$$

Additionally, the net guide index $N_{GI}(a_i)$ of a_i in connection with the other choice options is represented in this manner:

$$N_{GI}(a_i) = \sum_{i'=1, i' \neq i}^{m} GI(T_i, T_{i'}) - \sum_{i'=1, i' \neq i}^{m} GI(T_{i'}, T_i). \tag{23}$$

Obviously, a choice option a_i enjoys a greater preference with a higher $N_{SI}(a_i)$ and a higher $N_{SI}(a_i)$. Following the above rationale, the eventual choice option must satisfy the conditions that its net superiority identifier should be at a maximum and its net guide index at a maximum simultaneously. If both these conditions are not fulfilled, the alternative that possesses the highest average rank would be selected as an eventual solution. Furthermore, the complete-preference predominance ranks can be rendered by reranking choice options in accordance with the average ranks.

The initiated T-SF REGIME I and II methods for manipulating a multiple-criteria choice problem in uncertain contexts with T-spherical fuzziness are devised systematically as Algorithms I and II, respectively, described below. The two algorithms consist of five phases, including formulating a multiple-criteria choice problem in Phase (i), constructing score function-based superiority criteria in Phase (ii), ascertaining the superiority measurements in Phase (iii), pinpointing the REGIME matrix and guide indices in Phase (iv), and generating the eventual partial/complete ranking in Phase (v).

Algorithm I: T-SF REGIME I method

Phase (i): *Steps I.1–I.3*

Step I.1. Designate the collection of choice options $A = \{a_1, a_2, \dots, a_m\}$ and the collection of judgment criteria $C = \{c_1, c_2, \dots, c_n\}$.

Step I.2. Signify the normalized weight w_j in conjunction with each $c_j \in C$ and the T-SF evaluation value $t_{ij} = (\mu_{ij}, \eta_{ij}, \nu_{ij})$ along with the refusal membership γ_{ij}.

Step I.3. Compose the T-SF characteristic $T_i = \{\langle c_j, (\mu_{ij}, \eta_{ij}, \nu_{ij})\rangle \mid 0 \leqslant (\mu_{ij})^z + (\eta_{ij})^z + (\nu_{ij})^z \leqslant 1\ \forall c_j \in C\}$ in Eq. (7) associated with each $a_j \in A$ for $z \in Z^+$.

Phase (ii): *Steps I.4 and I.5*

Step I.4. Utilize Eqs. (6) and (5) to compute the score function $Sf(t_{ij})$ and the accuracy value $Av(t_{ij})$, respectively, of each T-SF evaluation value t_{ij} embedded in T_i.

Step I.5. Obtain the superiority criteria in which T_i is at least as good as $T_{i'}$ based on score functions and build the collection of superiority criteria $Cs(T_i, T_{i'})$ using Eq. (8).

Phase (iii): *Steps I.6 and I.7*

Step I.6. Compute the discrepancy between $Sf(t_{ij})$ and $Sf(t_{i'j})$ for any two choice options $a_i, a_{i'} \in A$ in terms of each superiority criterion $c_j \in Cs(T_i, T_{i'})$.

Step I.7. Make use of the normalized weight w_j to determine the superiority identifier $SI(T_i, T_{i'})$ in connection with a_i over $a_{i'}$ using Eq. (11).

Phase (iv): *Steps I.8 and I.9*

Step I.8. Apply Eq. (12) to generate the REGIME identifier $Ri(t_{ij}, t_{i'j})$ to establish the REGIME matrix Rm in Eq. (14) for all ordered couples of choice options.

Step I.9. Fuse the normalized weight w_j and the REGIME identifier $Ri(t_{ij}, t_{i'j})$ to derive the guide index $GI(T_i, T_{i'})$ relative to a_i over $a_{i'}$ using Eq. (15).

Phase (v): *Steps I.10–I.13*

Step I.10. Exploit Eq. (16) to gain an entry $Bs(a_i, a_{i'})$ based on superiority identifiers for building the superiority-based Boolean matrix Bs in Eq. (17).

Step I.11. Use Eq. (18) to acquire an entry $Bg(a_i, a_{i'})$ through the utility of guide indices for setting up the guide-based Boolean matrix Bg in Eq. (19).

Step I.12. Employ Eq. (20) to derive an entry $Bc(a_i, a_{i'})$ using a peer-to-peer multiplication operation for constructing the comprehensive Boolean matrix Bc in Eq. (21).

Step I.13. Confirm the partial-preference predominance rank regarding a_i over $a_{i'}$ if $Bc(a_i, a_{i'}) = 1$. Sketch a dominance graph for yielding evidently beneficial choice options.

Algorithm II: T-SF REGIME II method

Phases (i)–(iv): *Steps II.1–II.9*

Steps II.1–II.9. See Steps I.1–I.9 in Algorithm I.

Phase (v): *Steps II.10–II.12*

Step II.10. Derive the net superiority identifier $N_{SI}(a_i)$ of a_i relating to the other choice options using Eq. (22), and then rank all m options according to $N_{SI}(a_i)$ in descending order.

Step II.11. Delineate the net guide index $N_{GI}(a_i)$ of a_i with relevance to the other choice options using Eq. (23), and then rank all m options depending on $N_{GI}(a_i)$ in descending order.

Step II.12. Select the top-ranked option as the eventual solution if it enjoys the maximal net superiority identifier and the maximal net guide index, otherwise select an option having the highest average rank. The complete-preference predominance ranks are yielded by reranking options based on the average ranks.

In comparison with the prevailing decision-making methods predicated on a variety of aggregation operators, such as various multiple-criteria evaluation approaches in Ali *et al.* (2020), Chen *et al.* (2021), Garg *et al.* (2021), Guleria and Bajaj (2021), Khan *et al.* (2021b), Liu *et al.* (2021a), Liu *et al.* (2021b), Munir *et al.* (2020), and Munir *et al.* (2021), the T-SF REGIME architecture and techniques propounded by this research are simpler, easier to use, and more efficacious. The initiated T-SF REGIME methodology in this study possesses a comprehensive theoretical basis; but in terms of execution procedures, it is quite simple and conforms to human intuitive judgments. For analysts or decision makers, the principles of the REGIME method are easy to understand and accept. Moreover, abstract theoretical foundations can be realized through the developed notions and measurements, such as the superiority index, superiority identifier, REGIME identifier, and guide index. The decision maker can exploit systematic algorithms, i.e. T-SF REGIME I and II, to manage real-world multiple-criteria choice problems in highly complex and uncertain environments. In a nutshell, the most significant advantage of the propounded algorithms is highly intellectual but still easily understandable to use. The advanced methodology is designed to be easy for an untrained decision maker to manipulate, and it will facilitate the determination of eventual partial- and complete-preference predominance rankings of choice options in an uncomplicated and effectual manner.

4. Real-World Application and Comparisons

This section endeavors to study a real-life choice issue to demonstrate the technical practicality and strengths of using the T-SF REGIME methodology for multiple-criteria evaluations. Comparative discussions are also held under different score values for understanding the effectiveness and value of the initiated methods. Furthermore, this section makes more comparisons with the T-SF versions of other multiple-criteria evaluation methods to give substance to the superiority of the T-SF REGIME methodology. Consider that the TOPSIS (i.e. Technique for Order Preference by Similarity to Ideal Solutions) and the VIKOR (i.e. VIseKriterijumska Optimizacija I Kompromisno Resenje) are widely employed and reliable in resolving issues for multiple-criteria choice analysis. This section institutes certain comparisons of the propounded method with the T-SF TOPSIS and the T-SF VIKOR for stating the merits.

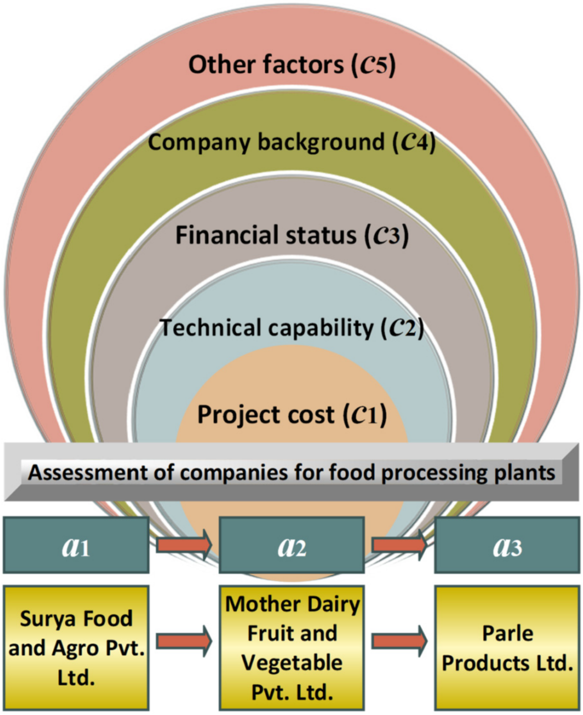


Fig. 2. The selection problem of companies for erecting food processing plants.

4.1. *Realistic Application and Discussions*

The investigated selection problem of companies for erecting food processing plants was originated from the case study in Garg *et al.* (2018), as outlined in Fig. 2. The Jharkhand government in India attempts to establish essential agricultural-focused industries in rural regions. The authorities constituted the global investor summit and encouraged companies and enterprises for the investment in the surrounding countryside. Moreover, the authorities made a formal public statement about adequate facilities that were exploitable to construct food processing plants in the countryside.

To begin with, the T-SF REGIME I method in Algorithm I would be illustrated with the realistic selection problem of companies for building food processing plants. As disclosed in Fig. 2, there are five judgment criteria (i.e. c_1–c_5) and three choice options (i.e. a_1–a_3) in this multiple-criteria choice task. In Step I.1 of Phase (i), the collection of choice options $A = \{a_1, a_2, a_3\}$ and the collection of judgment criteria $C = \{c_1, c_2, \dots, c_5\}$. Concerning Step I.2, as stated by Garg *et al.* (2018), the normalized weights in conjunction with the five criteria were derived by the agency of the normal distribution-based approach. In conformity with the T-SF evaluation value $t_{ij} = (\mu_{ij}, \eta_{ij}, \nu_{ij})$ in Garg *et al.* (2018), this study calculated the grade of refusal membership γ_{ij} corresponding to t_{ij}. The foregoing decision data related to each choice option are depicted in Table 1. In Step I.3, the parameter $z = 3$ for each t_{ij}, and the T-SF characteristic $T_i = \{\langle c_1, t_{i1}\rangle, \langle c_2, t_{i2}\rangle, \dots, \langle c_5, t_{i5}\rangle\} =$

Table 1
Decision information pertaining to each choice option.

		Choice option a_1		Choice option a_2		Choice option a_3	
c_j	w_j	$(\mu_{1j}, \eta_{1j}, \nu_{1j})$	γ_{1j}	$(\mu_{2j}, \eta_{2j}, \nu_{2j})$	γ_{2j}	$(\mu_{3j}, \eta_{3j}, \nu_{3j})$	γ_{3j}
c_1	0.1117	(0.7, 0.5, 0.6)	0.6811	(0.5, 0.4, 0.6)	0.8411	(0.4, 0.1, 0.2)	0.9750
c_2	0.2365	(0.9, 0.5, 0.4)	0.4344	(0.7, 0.2, 0.3)	0.8536	(0.5, 0.4, 0.1)	0.9322
c_3	0.3036	(0.4, 0.2, 0.1)	0.9750	(0.5, 0.3, 0.6)	0.8582	(0.0, 0.0, 0.5)	0.9565
c_4	0.2365	(0.5, 0.3, 0.4)	0.9221	(0.4, 0.1, 0.6)	0.8959	(0.6, 0.2, 0.2)	0.9158
c_5	0.1117	(0.6, 0.4, 0.5)	0.8411	(0.5, 0.2, 0.4)	0.9295	(0.6, 0.1, 0.5)	0.8698

Table 2
Results of score functions, accuracy values, and collections of superiority criteria.

	Choice option a_1		Choice option a_2		Choice option a_3	
c_j	$Sf(t_{1j})$	$Av(t_{1j})$	$Sf(t_{2j})$	$Av(t_{2j})$	$Sf(t_{3j})$	$Av(t_{3j})$
c_1	0.2128	0.6840	0.2207	0.4050	0.6842	0.0730
c_2	0.6035	0.9180	0.7629	0.3780	0.6107	0.1900
c_3	0.6842	0.0730	0.2864	0.3680	0.4335	0.1250
c_4	0.5626	0.2160	0.3013	0.2810	0.7449	0.2320
c_5	0.4272	0.4050	0.5977	0.1970	0.5416	0.3420
	$Cs(T_1, T_2) = \{c_3, c_4\}$		$Cs(T_2, T_1) = \{c_1, c_2, c_5\}$		$Cs(T_3, T_1) = \{c_1, c_2, c_4, c_5\}$	
	$Cs(T_1, T_3) = \{c_3\}$		$Cs(T_2, T_3) = \{c_2, c_5\}$		$Cs(T_3, T_2) = \{c_1, c_3, c_4\}$	

$\{\langle c_1, (\mu_{i1}, \eta_{i1}, \nu_{i1})\rangle, \langle c_2, (\mu_{i5}, \eta_{i5}, \nu_{i5})\rangle\}$ for each $a_i \in A$. It is worth pointing out that a weight-assessing approach sprung from normal distributions and launched by Xu (2005) was employed to generate the weights of five judgment criteria that fulfill the normalization requirement. On grounds of the notion of normal distributions, the mean and the standard deviation of the sequence $1, 2, \dots, 5$ were derived by $(1 + 5)/2 = 3$ and $\{[(1-3)^2 + (2-3)^2 + (3-3)^2 + (4-3)^2 + (5-3)^2]/5\}^{0.5} = 1.4142$, respectively. By virtue of Xu's (2005) approach, the weights were yielded along these lines:

$$\begin{aligned} w_1 = {} & e^{-[(1-3)^2/2(1.4142)^2]} / \big(e^{-[(1-3)^2/2(1.4142)^2]} + e^{-[(2-3)^2/2(1.4142)^2]} \\ & + e^{-[(3-3)^2/2(1.4142)^2]} + e^{-[(4-3)^2/2(1.4142)^2]} + e^{-[(5-3)^2/2(1.4142)^2]}\big) = 0.1117, \\ w_2 = {} & 0.2365, \quad w_3 = 0.3036, \quad w_4 = 0.2365, \quad \text{and} \quad w_5 = 0.1117. \end{aligned}$$

Consider Phase (ii) as an illustration. In Step I.4, the calculation consequences of the score function $Sf(t_{ij})$ and the accuracy value $Av(t_{ij})$ using Eqs. (6) and (5), respectively, are exhibited in the upper part of Table 2. Concerning Step I.5, this study exploited Eq. (8) to produce the collections of superiority criteria for paired T-SF characteristics, as displayed in the lower part of Table 2. Take the juxtaposition of the T-SF characteristics T_2 and T_3 as an example. As stated in Table 2, it was recognized that $Sf(t_{22}) > Sf(t_{32})$ and $Sf(t_{25}) > Sf(t_{35})$, which leads to $Cs(T_2, T_3) = \{c_2, c_5\}$. Additionally, it was realized that $Sf(t_{31}) > Sf(t_{21})$, $Sf(t_{33}) > Sf(t_{23})$, and $Sf(t_{34}) > Sf(t_{24})$, which brings about $Cs(T_3, T_2) = \{c_1, c_3, c_4\}$.

Table 3
Outcomes relevant to the score function-based superiority identifiers.

c_j	a_1 over a_2 $Sf(t_{1j})$–$Sf(t_{2j})$	a_1 over a_3 $Sf(t_{1j})$–$Sf(t_{3j})$	a_2 over a_1 $Sf(t_{2j})$–$Sf(t_{1j})$	a_2 over a_3 $Sf(t_{2j})$–$Sf(t_{3j})$	a_3 over a_1 $Sf(t_{3j})$–$Sf(t_{1j})$	a_3 over a_2 $Sf(t_{3j})$–$Sf(t_{2j})$
c_1	–	–	0.0079	–	0.4714	0.4636
c_2	–	–	0.1594	0.1522	0.0072	–
c_3	0.3978	0.2507	–	–	–	0.1471
c_4	0.2613	–	–	–	0.1824	0.4436
c_5	–	–	0.1705	0.0561	0.1144	–
	(T_1, T_2)	(T_1, T_3)	(T_2, T_1)	(T_2, T_3)	(T_3, T_1)	(T_3, T_2)
SI	0.1826	0.0761	0.0576	0.0423	0.1103	0.2014

Concerning Step I.6 in Phase (iii), the discrepancy between $Sf(t_{ij})$ and $Sf(t_{i'j})$ for each superiority criterion $c_j \in Cs(T_i, T_{i'})$ was received, as demonstrated in the upper part of Table 3. Relating to Step I.7, the weighted discrepancy between $Sf(t_{ij})$ and $Sf(t_{i'j})$ was aggregated across all $c_j \in Cs(T_i, T_{i'})$ to calculate the superiority identifier $SI(T_i, T_{i'})$ using Eq. (11), and the outcomes are presented in the lowest row of Table 3. Consider the pair (T_2, T_1) for instance. Based on $Cs(T_2, T_1) = \{c_1, c_2, c_5\}$, it was rendered that $SI(T_2, T_1) = \sum_{c_j \in \{c_1, c_2, c_5\}} w_j (Sf(t_{2j}) - Sf(t_{1j})) = 0.1117 \times 0.0079 + 0.2365 \times 0.1594 + 0.1117 \times 0.1705 = 0.0576$.

Relating to Step I.8 in Phase (iv), the REGIME identifier $Ri(t_{ij}, t_{i'j})$ was produced with the aid of $Sf(t_{ij})$ and $Sf(t_{i'j})$ using Eq. (12) to constitute the REGIME vector $Rv(T_i, T_{i'})$ for paired T-SF characteristics T_i and $T_{i'}$. Employing Eq. (14), the REGIME matrix Rm was constructed by taking together the REGIME vectors for all ordered pairs of $(a_i, a_{i'})$:

$$Rm = \begin{bmatrix} Rv(T_1, T_2) \\ Rv(T_1, T_3) \\ Rv(T_2, T_1) \\ Rv(T_2, T_3) \\ Rv(T_3, T_1) \\ Rv(T_3, T_2) \end{bmatrix} = \begin{array}{c} \\ (a_1, a_2) \\ (a_1, a_3) \\ (a_2, a_1) \\ (a_2, a_3) \\ (a_3, a_1) \\ (a_3, a_2) \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5 \end{array} \\ \begin{bmatrix} -1 & -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 \end{bmatrix} \end{array}.$$

Concerning Step I.9, this study exploited Eq. (15) to synthesize the normalized weight w_j and the REGIME identifier $Ri(t_{ij}, t_{i'j})$ for determining the guide index $GI(T_i, T_{i'})$ by: $GI(T_1, T_2) = \sum_{j=1}^{5} w_j \cdot Ri(t_{1j}, t_{2j}) = 0.1117 \cdot (-1) + 0.2365 \cdot (-1) + 0.3036 \cdot 1 + 0.2365 \cdot 1 + 0.1117 \cdot (-1) = 0.0802$, $GI(T_1, T_3) = -0.3928$, $GI(T_2, T_1) = -0.0802$, $GI(T_2, T_3) = -0.3036$, $GI(T_3, T_1) = 0.3928$, and $GI(T_3, T_2) = 0.3036$.

Considering Steps I.10 and I.11 in Phase (v), this paper made use of Eqs. (16) and (18) to acquire the entries $Bs(a_i, a_{i'})$ and $Bg(a_i, a_{i'})$, respectively, for $a_i, a_{i'} \in A$. Moreover, this paper employed Eqs. (17) and (19) to generate the superiority-based Boolean matrix Bs and the guide-based Boolean matrix Bg, respectively. In Step I.12, the entry $Bc(a_i, a_{i'})$ was delineated as $Bs(a_i, a_{i'}) \cdot Bg(a_i, a_{i'})$ according to Eq. (20) for organizing the compre-

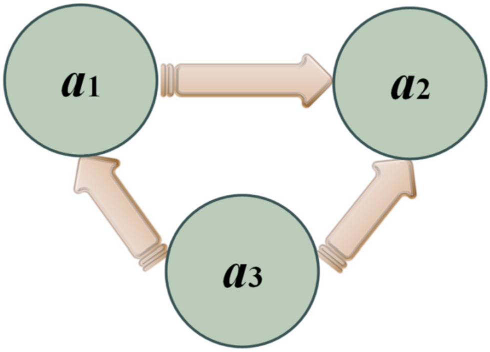


Fig. 3. The dominance graph yielded by the T-SF REGIME I prioritization procedure.

hensive Boolean matrix Bc in Eq. (21). The three Boolean matrices are as shown:

$$Bs = \begin{bmatrix} - & 1 & 0 \\ 0 & - & 0 \\ 1 & 1 & - \end{bmatrix}, \quad Bg = \begin{bmatrix} - & 1 & 0 \\ 0 & - & 0 \\ 1 & 1 & - \end{bmatrix}, \quad \text{and} \quad Bc = \begin{bmatrix} - & 1 & 0 \\ 0 & - & 0 \\ 1 & 1 & - \end{bmatrix}.$$

In Step I.13, the partial-preference predominance rankings $a_1 \succ^{\text{REGIMEI}} a_2$, $a_3 \succ^{\text{REGIMEI}} a_1$, and $a_3 \succ^{\text{REGIMEI}} a_2$ were generated by way of $Bc(a_1, a_2) = 1$, $Bc(a_3, a_1) = 1$, and $Bc(a_3, a_2) = 1$, respectively. On the report of the comprehensive Boolean matrix Bc, the dominance graph was portrayed in Fig. 3, which follows that a_3 (i.e. Parle Products Ltd.) was recognized as the most beneficial choice option.

On the flip-side, the T-SF REGIME II method in Algorithm II would be utilized to tackle the identical selection problem of companies for food processing plants. As mentioned previously, Steps II.1–II.9 are the same as Steps I.1–I.9. Concerning Step II.10 in Phase (v), this paper employed Eq. (22) to identify the net superiority identifiers in this manner: $N_{SI}(a_1) = (SI(T_1, T_2) + SI(T_1, T_3)) - (SI(T_2, T_1) + SI(T_3, T_1)) = (0.1826 + 0.0761) - (0.0576 + 0.1103) = 0.0908$, $N_{SI}(a_2) = -0.2840$, and $N_{SI}(a_3) = 0.1933$. The complete ranking $a_3 \succ^{SI} a_1 \succ^{SI} a_2$ was received in keeping with $N_{SI}(a_3) > N_{SI}(a_1) > N_{SI}(a_2)$. Next, relating to Step II.11, this study made use of Eq. (23) to attain the net guide indices in this fashion: $N_{GI}(a_1) = (GI(T_1, T_2) + GI(T_1, T_3)) - (GI(T_2, T_1) + GI(T_3, T_1)) = (0.0802 - 0.3928) - (-0.0802 + 0.3928) = -0.6252$, $N_{GI}(a_2) = -0.7676$, and $N_{GI}(a_3) = 1.3928$. The complete ranking $a_3 \succ^{GI} a_1 \succ^{GI} a_2$ was received in keeping with $N_{GI}(a_3) > N_{GI}(a_1) > N_{GI}(a_2)$.

Finally, in Step II.12, the complete-preference predominance ranking $a_3 \succ^{\text{REGIMEII}} a_1 \succ^{\text{REGIMEII}} a_2$ were rendered as supported by $a_3 \succ^{SI} a_1 \succ^{SI} a_2$ and $a_3 \succ^{GI} a_1 \succ^{GI} a_2$. Therefore, in the same vein, the top-ranked option a_3 was selected as the eventual solution. The resulting outcomes yielded by the prioritization procedures of T-SF REGIME I and II were identical to that used in the decision-aiding approach with the aid of geometric aggregation operations in Garg *et al.* (2018).

In an attempt to validate the methodological advantages, this paper implements a comparative approach to different score values in the initiated T-SF REGIME methods and

Table 4
Summary outcomes in the comparative analysis on grounds of score values.

	c_1	c_2	c_3	c_4	c_5
$Sv(t_{1j})$ for a_1	0.1270	0.6650	0.0630	0.0610	0.0910
$Sv(t_{2j})$ for a_2	−0.0910	0.3160	−0.0910	−0.1520	0.0610
$Sv(t_{3j})$ for a_3	0.0560	0.1240	−0.1250	0.2080	0.0910

	a_1 over a_2	a_1 over a_3	a_2 over a_1	a_2 over a_3	a_3 over a_1	a_3 over a_2
$SI(T_i, T_{i'})$	0.2074	0.1930	0.0000	0.0557	0.0348	0.1049
$Bs(a_i, a_{i'})$	1	1	0	0	0	1
$GI(T_i, T_{i'})$	1.0000	0.4153	−1.0000	0.0802	−0.4153	−0.0802
$Bg(a_i, a_{i'})$	1	1	0	0	0	1
$Bc(a_i, a_{i'})$	1	1	0	0	0	1

	Choice option a_1	Choice option a_2	Choice option a_3
$N_{SI}(a_i)$	0.3656	−0.2565	−0.1090
$N_{GI}(a_i)$	2.8306	−1.8396	−0.9910

techniques. Based on Definition 4, the score value $Sv(t_{ij})$ was exploited to replace the score function $Sf(t_{ij})$. Herein, it was received that $Sv(t_{ij}) = (\mu_{ij})^3 - (\nu_{ij})^3$ for each T-SF characteristic t_{ij} in the selection problem of companies. Table 4 reveals the summary consequences through the agency of the score value $Sv(t_{ij})$.

First, let us examine the results using the score value-based T-SF REGIME I method. As attested by the entry $Bc(a_i, a_{i'})$, the comparative study produced the partial-preference predominance rankings $a_1 \succ^{\text{REGIME I}} a_2$, $a_1 \succ^{\text{REGIME I}} a_3$, and $a_3 \succ^{\text{REGIME I}} a_2$ owing to $Bc(a_1, a_2) = 1$, $Bc(a_1, a_3) = 1$, and $Bc(a_3, a_2) = 1$, respectively. Next, consider the score value-based T-SF REGIME II method. Supported by $a_1 \succ^{SI} a_3 \succ^{SI} a_2$ and $a_1 \succ^{GI} a_3 \succ^{GI} a_2$, the comparative study generated the complete-preference predominance ranking $a_1 \succ^{\text{REGIME II}} a_3 \succ^{\text{REGIME II}} a_2$. Nevertheless, such outcomes were directly conflicting with the ultimate ranking $a_3 \succ a_1 \succ a_2$ yielded by Garg *et al.* (2018). Garg *et al.* (2018) corroborated that the choice option a_3 is superior to a_1; however, the comparative study delivered different consequences, i.e. the partial ordering $a_1 \succ^{\text{REGIME I}} a_3$ and the complete ordering $a_1 \succ^{\text{REGIME II}} a_3 \succ^{\text{REGIME II}} a_2$. These conflicting results were attributed to the use of the score value $Sv(t_{ij})$ in the T-SF REGIME I and II series of steps. Thus, the exploitation of the score function $Sf(t_{ij})$ has the ability to achieve more desirable solution consequences. The correctness and effectuality of the evolved methodology have been manifested through the medium of the comparative study and discussions.

The evolved T-SF REGIME methodology possesses an exceptional ability to make more accurate decisions by using a more suitable and efficacious measurement system concerning the characteristics of T-SF information. In particular, the initiated T-SF REGIME I and II prioritization procedures can make more precise decisions. Results are presented in the format of the partial-and complete-preference predominance ranks and can be utilized for the further applications in decision-making support systems.

4.2. *Comparisons with T-SF Versions of TOPSIS and VIKOR*

This subsection attempts to make comparisons with other decision-making approaches carefully and objectively to verify the performance of the evolved T-SF REGIME methodology. As is well known, the TOPSIS and VIKOR have been widely used throughout the compromise decision-making processes and have been renowned as anchor dependent models in multiple criteria analysis. From this basis, this subsection exploits the T-SF versions of TOPSIS and VIKOR to facilitate comparisons.

Choice options that come nearer to the positive-ideal option in separation measures are more favourable than those that come farther away. In contrast, choice options that keep away from the negative-ideal option via separation measures are more favourable than those that come near. The rationale of human choice is to simultaneously come near to the positive-ideal option and keep away from the negative-ideal option to the most possible extent. Such axioms of choice have explicitly assumed that there exist sharply bipolar anchor values of reference. Two types of bipolar anchor values were utilized in the comparative analysis with the T-SF TOPSIS and the T-SF VIKOR: fixed ideals and displaced ideals.

Take into consideration the selection problem of companies for erecting food processing plants. Place the fixed positive-ideal option a_+ and the fixed negative-ideal option a_-; their corresponding T-SF characteristics T_+ and T_- were expounded in this manner:

$$\begin{aligned}
T_+ &= \big\{\big\langle c_j, (\mu_{+j}, \eta_{+j}, \nu_{+j})\big\rangle \,\big|\, \forall c_j \in C\big\} \\
&= \big\{\big\langle c_1, (1, 0, 0)\big\rangle, \big\langle c_2, (1, 0, 0)\big\rangle, \dots, \big\langle c_5, (1, 0, 0)\big\rangle\big\}, \\
T_- &= \big\{\big\langle c_j, (\mu_{-j}, \eta_{-j}, \nu_{-j})\big\rangle \,\big|\, \forall c_j \in C\big\} \\
&= \big\{\big\langle c_1, (0, 0, 1)\big\rangle, \big\langle c_2, (0, 0, 1)\big\rangle, \dots, \big\langle c_5, (0, 0, 1)\big\rangle\big\}.
\end{aligned}$$

Place the displaced positive-ideal option a_* and the displaced negative-ideal option $a_\#$. On the grounds of the outcomes of score functions in Table 2, the largest values with respect to five judgment criteria were identified as follows: $Sf(t_{31}) = 0.6842$, $Sf(t_{22}) = 0.7629$, $Sf(t_{13}) = 0.6842$, $Sf(t_{34}) = 0.7449$, and $Sf(t_{25}) = 0.5977$ for $c_1, c_2, \dots, c_5$, respectively. Additionally, the smallest values of score functions with relevance to five judgment criteria were identified in this manner: $Sf(t_{11}) = 0.2128$, $Sf(t_{12}) = 0.6035$, $Sf(t_{23}) = 0.2864$, $Sf(t_{24}) = 0.3013$, and $Sf(t_{15}) = 0.4272$ for $c_1, c_2, \dots, c_5$, respectively. As a direct consequence, the T-SF characteristics $T_* = \{\langle c_j, (\mu_{*j}, \eta_{*j}, \nu_{*j})\rangle | \forall c_j \in C\}$ and $T_\# = \{\langle c_j, (\mu_{\#j}, \eta_{\#j}, \nu_{\#j})\rangle \mid \forall c_j \in C\}$ associated with a_* and $a_\#$, respectively, were delineated in this fashion:

$$\begin{aligned}
T_* = \big\{&\big\langle c_1, (0.4, 0.1, 0.2)\big\rangle, \big\langle c_2, (0.7, 0.2, 0.3)\big\rangle, \big\langle c_3, (0.4, 0.2, 0.1)\big\rangle, \\
&\big\langle c_4, (0.6, 0.2, 0.2)\big\rangle, \big\langle c_5, (0.5, 0.2, 0.4)\big\rangle\big\}, \\
T_\# = \big\{&\big\langle c_1, (0.7, 0.5, 0.6)\big\rangle, \big\langle c_2, (0.9, 0.5, 0.4)\big\rangle, \big\langle c_3, (0.5, 0.3, 0.6)\big\rangle, \\
&\big\langle c_4, (0.4, 0.1, 0.6)\big\rangle, \big\langle c_5, (0.6, 0.4, 0.5)\big\rangle\big\}.
\end{aligned}$$

Table 5
Results of the closeness coefficients yielded by the T-SF TOPSIS approach.

	$\chi=1$	$\chi=2$	$\chi=3$	$\chi=4$	$\chi=5$	$\chi=6$	$\chi=7$	$\chi=8$	$\chi=9$	$\chi=10$	$\chi\to\infty$
Outcomes based on bipolar anchor values of the fixed ideals											
$CC_+^\chi(a_1)$	0.7259	0.7750	0.7873	0.7915	0.7931	0.7939	0.7943	0.7945	0.7946	0.7946	0.7946
$CC_+^\chi(a_2)$	0.7448	0.7978	0.8131	0.8197	0.8231	0.8249	0.8260	0.8267	0.8271	0.8274	0.8279
$CC_+^\chi(a_3)$	0.8270	0.8761	0.8884	0.8932	0.8955	0.8968	0.8976	0.8981	0.8985	0.8987	0.8997
Outcomes based on bipolar anchor values of the displaced ideals											
$CC_*^\chi(a_1)$	0.2328	0.2685	0.2823	0.2886	0.2918	0.2935	0.2945	0.2951	0.2954	0.2956	0.2960
$CC_*^\chi(a_2)$	0.3412	0.3806	0.3968	0.4051	0.4098	0.4125	0.4142	0.4152	0.4159	0.4163	0.4171
$CC_*^\chi(a_3)$	0.6449	0.6902	0.7041	0.7096	0.7122	0.7137	0.7146	0.7153	0.7158	0.7162	0.7179

First, give consideration to the determination outcomes yielded by the T-SF TOPSIS method. Place a distance parameter $\chi \in Z^+$. By exploiting the Minkowski distance unfolded by Ju *et al.* (2021), the weighted distance between the T-SF characteristics T_i and $T_{i'}$ can be derived along these lines:

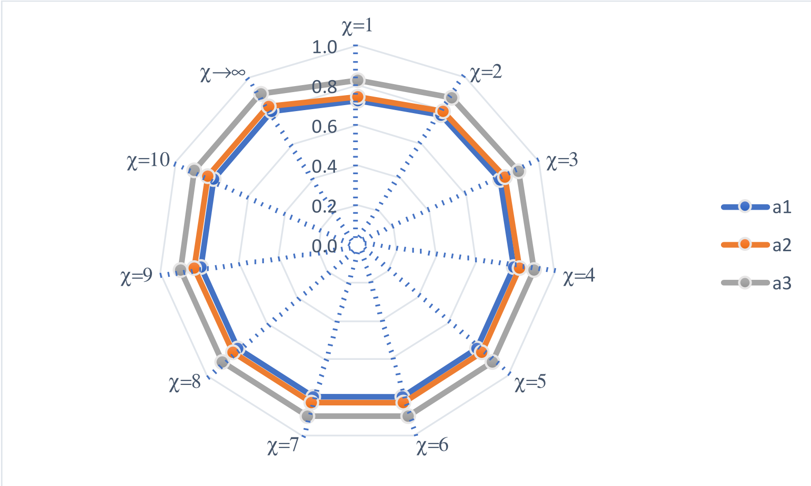
$$WD^\chi(T_i, T_{i'}) = \sum_{j=1}^{n} w_j \sqrt[\chi]{\frac{1}{2}\left(\left|(\mu_{ij})^t - (\mu_{i'j})^t\right|^\chi + \left|(\eta_{ij})^t - (\eta_{i'j})^t\right|^\chi + \left|(\nu_{ij})^t - (\nu_{i'j})^t\right|^\chi\right)}. \tag{24}$$

The closeness coefficients $CC_+^\chi(a_i)$ and $CC_*^\chi(a_i)$ of each choice option $a_i \in A$ on the grounds of the fixed and displaced ideal options, respectively, can be explicated in this manner:

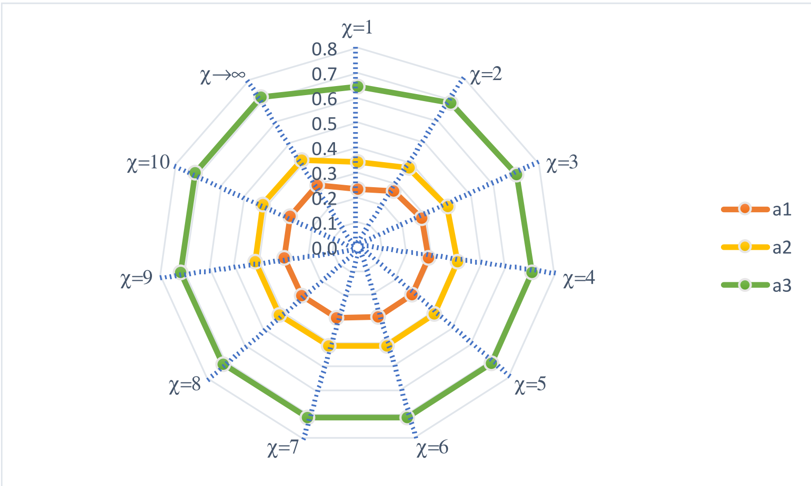
$$CC_+^\chi(a_i) = \frac{WD^\chi(T_i, T_-)}{WD^\chi(T_i, T_+) + WD^\chi(T_i, T_-)}, \tag{25}$$

$$CC_*^\chi(a_i) = \frac{WD^\chi(T_i, T_\#)}{WD^\chi(T_i, T_*) + WD^\chi(T_i, T_\#)}. \tag{26}$$

Based around the data in the selection problem of companies, the computation outcomes of the closeness coefficients $CC_+^\chi(a_i)$ and $CC_*^\chi(a_i)$ are exhibited in Table 5. Moreover, the contrasts of the resulting $CC_+^\chi(a_i)$ and $CC_*^\chi(a_i)$ are sketched in Fig. 4(a) and (b), respectively. It should be noted that the consistent predominance rankings $a_3 \succ^{\text{TOPSIS}_+} a_2 \succ^{\text{TOPSIS}_+} a_1$ and $a_3 \succ^{\text{TOPSIS}_*} a_2 \succ^{\text{TOPSIS}_*} a_1$ were acquired in cases that $\chi = 1, 2, \ldots, 10$ and $\chi \to \infty$ regardless of the exploitation of fixed ideals and displaced ideals. However, such a ranking (i.e. $a_3 \succ a_2 \succ a_1$) had a conflicting partial ordering in comparison with the ultimate ranking $a_3 \succ a_1 \succ a_2$ rendered by Garg *et al.* (2018), which indicates unreliability and untrustworthiness of the consequences rendered by the T-SF TOPSIS method predicated on the fixed and displaced ideal options. Contrary to the TOPSIS results, the propounded T-SF REGIME methodology can generate reasonable and convincing outcomes in preference predominance rankings of choice options. No matter the partial-preference rankings $a_1 \succ^{\text{REGIME I}} a_2$, $a_3 \succ^{\text{REGIME I}} a_1$, and



(a) Contrast of $CC_{+}^{\chi}(a_i)$



(b) Contrast of $CC_{*}^{\chi}(a_i)$

Fig. 4. The contrasting effect of the obtained closeness coefficients in different χ values.

$a_3 \succ^{\text{REGIME I}} a_2$ or the complete-preference ranking $a_3 \succ^{\text{REGIME II}} a_1 \succ^{\text{REGIME II}} a_2$ were consistent with the yielded results by Garg *et al.* (2018). The findings of the comparative study have given substance to the superiority of the T-SF REGIME approach over the T-SF TOPSIS method.

Next, consider the determination outcomes produced by the T-SF VIKOR method. Place any two T-SF evaluation values $t_{ij} = (\mu_{ij}, \eta_{ij}, \nu_{ij})$ and $t_{i'j} = (\mu_{i'j}, \eta_{i'j}, \nu_{i'j})$ in a specific multiple-criteria choice issue. The Minkowski distance presented by Ju *et al.* (2021) can be utilized to delineate two measurements about group utility and individual regret with the intention of forming a joint measure for compromise rankings. The weighted distance between t_{ij} and $t_{i'j}$ can be computed in this fashion:

$$D^{\chi}(t_{ij}, t_{i'j}) = \sqrt[\chi]{\frac{1}{2}\left(\left|(\mu_{ij})^t - (\mu_{i'j})^t\right|^{\chi} + \left|(\eta_{ij})^t - (\eta_{i'j})^t\right|^{\chi} + \left|(\nu_{ij})^t - (\nu_{i'j})^t\right|^{\chi}\right)}. \tag{27}$$

By incorporating the normalized weight w_j into the relative ratio of the distances $D^{\chi}(t_{ij}, t_{+j})$ to $D^{\chi}(t_{+j}, t_{-j})$, the group utility measure $Gu_{+}^{\chi}(T_i)$ and the individual regret measure $Ir_{+}^{\chi}(T_i)$ of the T-SF characteristic $T_i = \{\langle c_j, (\mu_{ij}, \eta_{ij}, \nu_{ij})\rangle \mid \forall c_j \in C\}$ based on the fixed ideals are manifested as follows:

$$Gu_{+}^{\chi}(T_i) = \sum_{j=1}^{n}\left(w_j \frac{D^{\chi}(t_{ij}, t_{+j})}{D^{\chi}(t_{+j}, t_{-j})}\right), \tag{28}$$

$$Ir_{+}^{\chi}(T_i) = \max_{j=1}^{n}\left(w_j \frac{D^{\chi}(t_{ij}, t_{+j})}{D^{\chi}(t_{+j}, t_{-j})}\right). \tag{29}$$

In an analogous way, the measures $Gu_{*}^{\chi}(T_i)$ and $Ir_{*}^{\chi}(T_i)$ based on the displaced ideals are given by:

$$Gu_{*}^{\chi}(T_i) = \sum_{j=1}^{n}\left(w_j \frac{D^{\chi}(t_{ij}, t_{*j})}{D^{\chi}(t_{*j}, t_{\#j})}\right), \tag{30}$$

$$Ir_{*}^{\chi}(T_i) = \max_{j=1}^{n}\left(w_j \frac{D^{\chi}(t_{ij}, t_{*j})}{D^{\chi}(t_{*j}, t_{\#j})}\right). \tag{31}$$

By virtue of a VIKOR parameter $\iota \in [0, 1]$, the joint measures $Jm_{+}^{\chi,\iota}(a_i)$ and $Jm_{*}^{\chi,\iota}(a_i)$ of each choice option a_i are elucidated in this manner:

$$\begin{aligned} Jm_{+}^{\chi,\iota}(a_i) = {} & \iota \times \frac{Gu_{+}^{\chi}(T_i) - \min_{i'=1}^{m} Gu_{+}^{\chi}(T_{i'})}{\max_{i'=1}^{m} Gu_{+}^{\chi}(T_{i'}) - \min_{i'=1}^{m} Gu_{+}^{\chi}(T_{i'})} \\ & + (1-\iota)\frac{Ir_{+}^{\chi}(T_i) - \min_{i'=1}^{m} Ir_{+}^{\chi}(T_{i'})}{\max_{i'=1}^{m} Ir_{+}^{\chi}(T_{i'}) - \min_{i'=1}^{m} Ir_{+}^{\chi}(T_{i'})}, \end{aligned} \tag{32}$$

$$\begin{aligned} Jm_{*}^{\chi,\iota}(a_i) = {} & \iota \times \frac{Gu_{*}^{\chi}(T_i) - \min_{i'=1}^{m} Gu_{*}^{\chi}(T_{i'})}{\max_{i'=1}^{m} Gu_{*}^{\chi}(T_{i'}) - \min_{i'=1}^{m} Gu_{*}^{\chi}(T_{i'})} \\ & + (1-\iota)\frac{Ir_{*}^{\chi}(T_i) - \min_{i'=1}^{m} Ir_{*}^{\chi}(T_{i'})}{\max_{i'=1}^{m} Ir_{*}^{\chi}(T_{i'}) - \min_{i'=1}^{m} Ir_{*}^{\chi}(T_{i'})}. \end{aligned} \tag{33}$$

Giving consideration to the selection problem of companies, the determined outcomes of the measures $Gu_{+}^{\chi}(T_i)$, $Ir_{+}^{\chi}(T_i)$, $Gu_{*}^{\chi}(T_i)$, and $Ir_{*}^{\chi}(T_i)$ are indicated in Table 6. Letting the VIKOR parameter $\chi = 0.5$, the resulting joint measures $Jm_{+}^{\chi,0.5}(a_i)$ and $Jm_{*}^{\chi,0.5}(a_i)$ are also revealed in this table. First, consider the bipolar anchor values using the fixed ideals. Two predominance rankings were produced in compliance with an increasing or-

Table 6
Results of the measurements related to the T-SF VIKOR approach.

	$\chi=1$	$\chi=2$	$\chi=3$	$\chi=4$	$\chi=5$	$\chi=6$	$\chi=7$	$\chi=8$	$\chi=9$	$\chi=10$	$\chi\to\infty$
Outcomes based on bipolar anchor values of the fixed ideals											
$Gu_+^\chi(T_1)$	0.4222	0.5172	0.5712	0.6031	0.6237	0.6381	0.6487	0.6567	0.6631	0.6682	0.7162
$Gu_+^\chi(T_2)$	0.5053	0.6048	0.6672	0.7050	0.7295	0.7465	0.7589	0.7683	0.7758	0.7818	0.8379
$Gu_+^\chi(T_3)$	0.4801	0.6308	0.7051	0.7468	0.7731	0.7912	0.8044	0.8144	0.8222	0.8286	0.8881
$Ir_+^\chi(T_1)$	0.1435	0.2009	0.2255	0.2390	0.2474	0.2532	0.2574	0.2606	0.2631	0.2651	0.2842
$Ir_+^\chi(T_2)$	0.1697	0.1936	0.2119	0.2236	0.2313	0.2367	0.2406	0.2436	0.2460	0.2479	0.2657
$Ir_+^\chi(T_3)$	0.1708	0.2163	0.2411	0.2553	0.2643	0.2705	0.2750	0.2784	0.2811	0.2833	0.3036
$Jm_+^{\chi,0.5}(a_1)$	0.0000	0.1619	0.2335	0.2422	0.2437	0.2439	0.2440	0.2440	0.2440	0.2440	0.2440
$Jm_+^{\chi,0.5}(a_2)$	0.9806	0.3857	0.3584	0.3545	0.3540	0.3539	0.3540	0.3540	0.3540	0.3540	0.3540
$Jm_+^{\chi,0.5}(a_3)$	0.8487	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Outcomes based on bipolar anchor values of the displaced ideals											
$Gu_*^\chi(T_1)$	0.5669	0.5595	0.5596	0.5604	0.5612	0.5618	0.5623	0.5626	0.5629	0.5630	0.5634
$Gu_*^\chi(T_2)$	0.6008	0.6084	0.6141	0.6177	0.6198	0.6211	0.6219	0.6224	0.6227	0.6229	0.6234
$Gu_*^\chi(T_3)$	0.4185	0.4212	0.4203	0.4199	0.4199	0.4200	0.4201	0.4202	0.4203	0.4203	0.4204
$Ir_*^\chi(T_1)$	0.2365	0.2365	0.2365	0.2365	0.2365	0.2365	0.2365	0.2365	0.2365	0.2365	0.2365
$Ir_*^\chi(T_2)$	0.3036	0.3036	0.3036	0.3036	0.3036	0.3036	0.3036	0.3036	0.3036	0.3036	0.3036
$Ir_*^\chi(T_3)$	0.2017	0.1892	0.1814	0.1778	0.1763	0.1756	0.1753	0.1752	0.1751	0.1751	0.1751
$Jm_*^{\chi,0.5}(a_1)$	0.5777	0.5763	0.5848	0.5885	0.5899	0.5905	0.5907	0.5908	0.5909	0.5910	0.5911
$Jm_*^{\chi,0.5}(a_2)$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$Jm_*^{\chi,0.5}(a_3)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

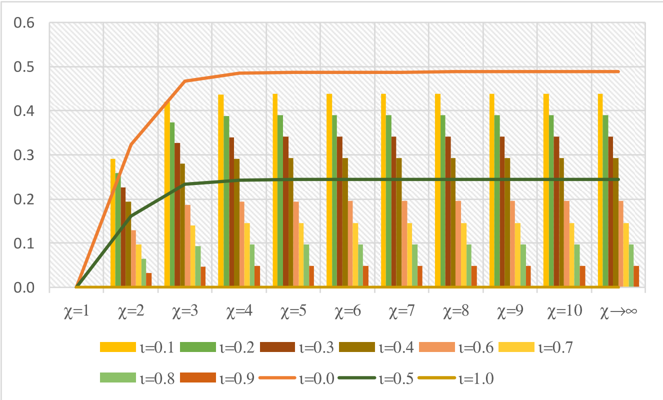
der of the $Jm_+^{\chi,0.5}(a_i)$ values, namely $a_1 \succ^{\mathrm{VIKOR}_+} a_3 \succ^{\mathrm{VIKOR}_+} a_2$, when $\chi = 1$, and $a_1 \succ^{\mathrm{VIKOR}_+} a_2 \succ^{\mathrm{VIKOR}_+} a_3$, when $\chi = 2, 3, \dots, 10$ and $\chi \to \infty$. Nevertheless, such consequences were strikingly different from the ultimate ranking $a_3 \succ a_1 \succ a_2$ resolved by Garg *et al.* (2018) and the proposed T-SF REGIME II algorithm. To be specific, the predominance relationship between a_1 and a_2 (i.e. $a_1 \succ a_2$) was generally concordant with the consequences in Garg *et al.* (2018) and the T-SF REGIME methodology. In particular, the choice option a_3 was the best solution in the results yielded by Garg *et al.* (2018) and the T-SF REGIME I and II algorithms; however, the T-SF VIKOR method pointed out that a_3 was the worst solution in cases that $\chi = 2, 3, \dots, 10$ and $\chi \to \infty$ from the perspective of the fixed ideals. Thus, based on bipolar anchor values of the fixed ideals, the T-SF VIKOR approach gave rise to disbelieving and unreasonable ranking outcomes. Next, making allowance for the bipolar anchor values using the displaced ideals, a reasonable and consistent ranking $a_3 \succ^{\mathrm{VIKOR}_*} a_1 \succ^{\mathrm{VIKOR}_*} a_2$ was obtained in keeping with the ascending order of the $Jm_*^{\chi,0.5}(a_i)$ values when $\chi = 1, 2, \dots, 10$ and $\chi \to \infty$. By comparison, the consequence judging by the $Jm_*^{\chi,0.5}(a_i)$ values were more believable and creditable than that by the $Jm_+^{\chi,0.5}(a_i)$ values. Even so, the results of applying the T-SF VIKOR method were still disputable and confusing for the decision maker because the obtained ranking outcomes would have great differences. The reason was that the contrasts of the obtained joint measures may be unstable and controversial based on distinct bipolar anchor values. In contrast, the T-SF REGIME methodology can deliver steady and well supported solution results, which demonstrates the strong points of the current approach.

In order to observe the changes of the joint measures $Jm_{+}^{\chi,\iota}(a_i)$ and $Jm_{*}^{\chi,\iota}(a_i)$ under different settings of the VIKOR parameter ι, this study designated the VIKOR parameter as $\iota = 0.0, 0.1, \ldots, 1.0$. First, make allowance for the bipolar anchor values using the fixed ideals. It should be noted that $Jm_{+}^{\chi,\iota}(a_3) = 1$ in every parameter combination of $\iota = 0.0, 0.1, \ldots, 1.0$ and $\chi = 2, 3, \ldots, 10$ and $\chi \to \infty$ because $\max_{i'=1}^{3} Gu_{+}^{\chi}(T_{i'}) = Gu_{+}^{\chi}(T_3)$ and $\max_{i'=1}^{3} Ir_{+}^{\chi}(T_{i'}) = Ir_{+}^{\chi}(T_3)$. Thus, it was recognized that $Jm_{+}^{\chi,\iota}(a_3) = \iota \times 1 + (1-\iota) \times 1 = 1$ in such scenarios. In this regard, it would merely have a look at the changes of the values of $Jm_{+}^{\chi,\iota}(a_1)$ and $Jm_{+}^{\chi,\iota}(a_2)$ in various settings, as portrayed in Fig. 5(a) and (b), respectively. On the flip-side, it would give consideration to the bipolar anchor values using the displaced ideals. In an analogous way, it was known that $Jm_{*}^{\chi,\iota}(a_2) = 1$ in every parameter combination of $\iota = 0.0, 0.1, \ldots, 1.0$ and $\chi = 1, 2, \ldots, 10$ and $\chi \to \infty$. Over and above that, it was found that $Jm_{*}^{\chi,\iota}(a_3) = 0$ because $\min_{i'=1}^{3} Gu_{*}^{\chi}(T_{i'}) = Gu_{*}^{\chi}(T_3)$ and $\min_{i'=1}^{3} Ir_{*}^{\chi}(T_{i'}) = Ir_{*}^{\chi}(T_3)$ in all scenarios. On the grounds of this, $Jm_{*}^{\chi,\iota}(a_3) = \iota \times 0 + (1-\iota) \times 0 = 0$ no matter which combination of parameters ι and χ. From this basis, the change of the $Jm_{*}^{\chi,\iota}(a_1)$ value was explored and depicted in Fig. 5(c). As sketched in Fig. 5, three special cases of $\iota = 0.0, 0.5, 1.0$ were represented using line charts, whereas the other cases were expressed by bar graphs. As revealed in Fig. 5, the values of the joint measures $Jm_{+}^{\chi,\iota}(a_1)$, $Jm_{+}^{\chi,\iota}(a_2)$, and $Jm_{*}^{\chi,\iota}(a_1)$ were deeply affected by the parameters ι and χ. Thus, such values were not stable, which may bring about different final rankings of the choice options. In contrast to the T-SF VIKOR technique, the evolved T-SF REGIME methodology can render more reliable and convincing results, which can make the final decision easy to implement.

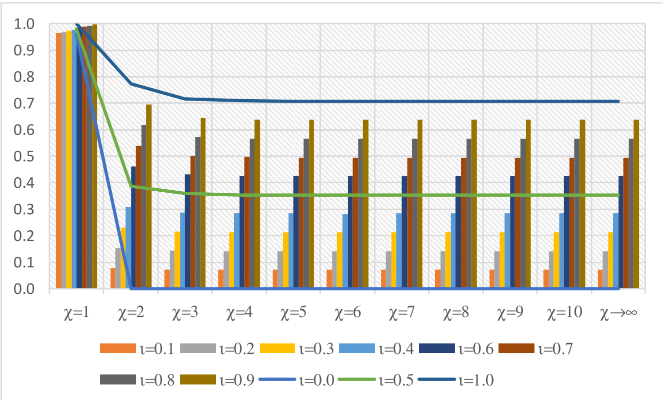
5. Conclusions

The T-SF theory has the productive competence and technological capability to manipulate ambiguous and equivocal decision information in a complex real-world environment. In particular, on the subject of fuzzy community, the T-SF model contains a comprehensive account of several beneficial non-standard fuzzy configurations, such as intuitionistic fuzziness, Pythagorean fuzziness, Fermatean fuzziness, q-rung orthopair fuzziness, picture fuzziness, and spherical fuzziness. The three motivations were the major driving forces to propound the T-SF REGIME methodology, consisting of: (1) high demand for utilizing the T-SF theory in specialized decision analysis, (2) technical gap of the current REGIME methods and techniques, and (3) lack of T-SF versions of the REGIME framework.

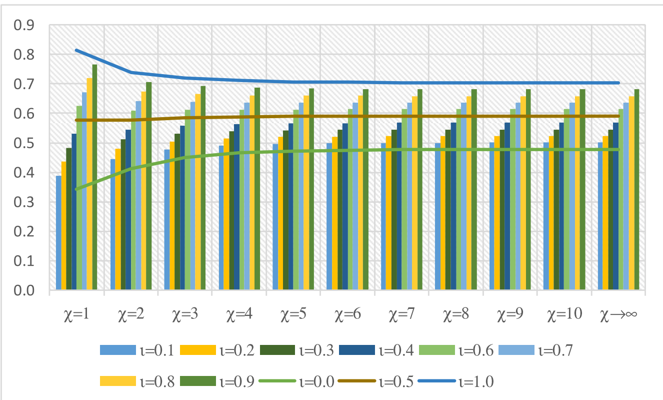
This study has made some noteworthy academic contributions to decision-making practice under complicated uncertainties, including the advancement of the REGIME-based technique, the beneficial measurement system in T-SF settings, superiority indices and identifiers for relative attractiveness, REGIME identifiers and guide indices for relative fittingness, and the efficacious T-SF REGIME I and II procedures for decision support. Consider that the REGIME method is a well-established approach to a multiple-criteria evaluation process and choice analysis; however, fresh enrichments of REGIME-based



(a) Contrast of $Jm_{+}^{\chi,\iota}(a_1)$



(b) Contrast of $Jm_{+}^{\chi,\iota}(a_2)$



(c) Contrast of $Jm_{*}^{\chi,\iota}(a_1)$

Fig. 5. Selected comparisons of the joint measures in every combination of parameters ι and χ.

techniques have not been discussed in T-SF decision situations. In this regard, this paper has unfolded a novel and creative T-SF REGIME methodology through the utility of an advisable measurement system in T-SF uncertain circumstances. This paper has utilized an efficacious score function to differentiate T-SF information in a thorough manner. Moreover, the superiority criteria have been recognized judging by score functions and accuracy values; the resulting outcomes have been used to determine the superiority index and the superiority identifier for ascertaining the relative attractiveness between the T-SF characteristics. Furthermore, this paper has identified the REGIME identifier and the REGIME vector to constitute the REGIME matrix and the guide index for ascertaining the relative fittingness between T-SF characteristics. Two effectual prioritization procedures have been inaugurated to generate the T-SF REGIME I and II predominance rankings regarding all choice options. The core notions in the T-SF REGIME I mechanism contain the superiority-based Boolean matrix, guide-based Boolean matrix, and comprehensive Boolean matrix. The core notions in the T-SF REGIME II mechanism involve the net superiority identifier and net guide index. The investigation toward the selection problem of companies for constructing food processing plants has exhibited the usefulness and superior points of employing the advanced T-SF REGIME methodology in tackling pragmatic decision issues.

However, the proposed T-SF REGIME methodology is subject to one major theoretical limitation. This paper has made an important contribution by strengthening the REGIME method aiming to empower it to manipulate T-FS uncertain information. The initiated T-FS version of the REGIME method has been validated to be an advantageous approach to multiple-criteria choice analysis within intricate equivocal environments. The technological applicability has been also illustrated in the selection problem of companies for erecting food processing plants. Nevertheless, decision makers or analysts might recognize some of the research limitations on the propounded methodology. The major limitation of the developed techniques is the early defuzzification of T-SF evaluation values by way of the score function advanced by Zeng *et al.* (2019). After constructing T-SF characteristics in the first phase of the T-SF REGIME framework, the T-SF evaluation values are then converted to the scalar value using the score function; moreover, the sequential phases are performed by the agency of the exploitation of such crisp forms. The specification of score functions can exploit enough information contained in T-SF evaluation values because of the utilization of grades of positive, neutral, negative, and refusal memberships. However, the T-SF evaluation values are defuzzied in a very early stage, which might make the T-SF REGIME method questionable in the process of evaluation, because perhaps certain information might be lost in practicality. In this regard, the initiated Algorithms I and II might result in a small amount of information not being considered in addressing a multiple-criteria choice problem in uncertain contexts, which is the main research limitation faced by the T-SF REGIME methodology.

Another consideration should be mentioned concerning the application of T-SF sets, namely, the data collection issue. The T-SF theory has been progressively concerned owing to its great ability for treating much complicated and obscure decision situation. However, a critical challenge for the T-SF theory to put into practice is how to collect evaluations from the decision maker and convert them into T-SF information. In most cases, the decision maker may be not familiar with the notion of T-SF sets. Thus, the decision maker may not know what T-SF evaluation values mean. Accordingly, the data collection process would be very challenging in case an analyst would like to collect evaluations from the decision maker. Therefore, how to reasonably produce T-SF evaluation values originating out of the decision maker's discernments and appraisals is an issue worthy of mention. This study suggests an approach to generating T-SF evaluation values by employing a linguistic rating system, i.e. using the scale for the linguistic variables. For example, Mathew *et al.* (2020) presented two nine-point rating scales and an eleven-point rating scale that can be exploited by decision makers to quantify their subjective judgments and assessments. By the same token, Farrokhizadeh *et al.* (2021) and Jin *et al.* (2021) also provided useful nine-point rating scales for facilitating data collection. These linguistic variables can be easily converted into spherical fuzzy or T-SF numbers. With the advent of linguistic rating scales involving spherical fuzziness, linguistic terms can be quantified by transforming them to spherical fuzzy numbers, which leads to convenient construction of T-SF evaluation values.

Furthermore, this paper proposes some future research work directions that are promising and appropriate. First, the proposed notions and measurements (e.g. the superiority index, superiority identifier, REGIME identifier, and guide index) can furnish theoretical bases to create other decision-making models and support mechanisms. For example, by expanding Garg and Rani's (2021) intuitionistic fuzzy MULTIMOORA (i.e. MULTIplicative form for Multiple Objective Optimization based on a Ratio Analysis) technique, our proposals can be incorporated into the MULTIMOORA scheme to adapt to T-SF decision contexts. By the same token, the proposed measurements can be exploited in the procedure of the ORESTE (i.e. Organisation, Rangement Et Synthèse de données relaTionnElles) approach for facilitating an advancement of ORESTE in T-SF circumstances. Secondly, in addition to multiple-criteria choice analysis, the procedures for the T-SF REGIME I and II prioritization can be recognized as a significant enhancement tool for exploring group decision analyses, sorting approaches, design and evaluation strategies, etc. By way of illustration, the sine-trigonometric operations presented by Garg (2021b) can be generalized to T-SF environments and then combined in the T-SF REGIME mechanism for group decision-making processes. Finally, the evolved T-SF REGIME methodology can be continuously modified and developed for adapting to different decision circumstances. To give an instance, Garg's (2021a) initiated possibility degree measure derived from the q-rung orthopair fuzzy model is suggested to be exploited and extended in the evolved T-SF REGIME methodology. These improvements and modifications can offer the most appropriate and functional approaches regarding the exact engineering, management science, economics, and business problem.

Acknowledgements

The author acknowledges the assistance of the respected editor and the anonymous referees for their insightful and constructive comments, which helped to improve the overall quality of the paper.

Funding

The author is grateful for grant funding support from the Ministry of Science and Technology, Taiwan (MOST 110-2410-H-182-005) and Chang Gung Memorial Hospital, Linkou, Taiwan (BMRP 574) during the completion of this study.

References

Akgün, A.A., Baycan, T., Nijkamp, P. (2015). Rethinking on sustainable rural development. *European Planning Studies*, 23(4), 678–692. https://doi.org/10.1080/09654313.2014.945813.

Akgün, A.A., van Leeuwen, E., Nijkamp, P. (2012). A multi-actor multi-criteria scenario analysis of regional sustainable resource policy. *Ecological Economics*, 78(June), 19–28. https://doi.org/10.1016/j.ecolecon.2012.02.026.

Akram, M., Naz, S., Shahzadi, S., Ziaa, F. (2021a). Geometric-arithmetic energy and atom bond connectivity energy of dual hesitant q-rung orthopair fuzzy graphs. *Journal of Intelligent & Fuzzy Systems*, 40(1), 1287–1307. https://doi.org/10.3233/JIFS-201605.

Akram, M., Shahzadi, G., Shahzadi, S. (2021b). Protraction of Einstein operators for decision-making under q-rung orthopair fuzzy model. *Journal of Intelligent & Fuzzy Systems*, 40(3), 4779–4798. https://doi.org/10.3233/JIFS-201611.

Ali, Z., Mahmood, T., Yang, M.-S. (2020). Complex T-spherical fuzzy aggregation operators with application to multi-attribute decision making. *Symmetry*, 12(8), 1311. https://doi.org/10.3390/sym12081311.

Alinezhad, A., Khalili, J. (2019). *New methods and applications in multiple attribute decision making (MADM)*. International Series in Operations Research & Management Science*: Vol.* 277. Springer, Berlin. https://doi.org/10.1007/978-3-030-15009-9.

Alipour, M., Hafezi, R., Rani, P., Hafezi, M., Mardani, A. (2021). A new Pythagorean fuzzy-based decision-making method through entropy measure for fuel cell and hydrogen components supplier selection. *Energy*, 234(November). Article ID 121208, 14 pp. https://doi.org/10.1016/j.energy.2021.121208.

Ashraf, S., Abdullah, S. (2021). Decision aid modeling based on sine trigonometric spherical fuzzy aggregation information. *Soft Computing*, 25(13), 8549–8572. https://doi.org/10.1007/s00500-021-05712-6.

Asgharizadeh, E., Safari, H., Faraji, Z., Majidian, S. (2014). Supply chain performance measurement by combining criterions of SCOR, Gunasekaran and BSC models with REGIME technique. *Journal of Basic Applied Scientific Research*, 4(3), 309–320.

Aspen, D.M., Sparrevik, M., Fet, A.M. (2015). Review of methods for sustainability appraisals in ship acquisition. *Environment Systems and Decisions*, 35(3), 323–333. https://doi.org/10.1007/s10669-015-9561-6.

Atanassov, K.T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3.

Briamonte, L., Pergamo, R., Arru, B., Furesi, R., Pulina, P., Madau, F.A. (2021). Sustainability goals and firm behaviours: a multi-criteria approach on Italian agro-food sector. *Sustainability (Switzerland)*, 13(10). Article ID 5589, 16 pp. https://doi.org/10.3390/su13105589.

Chen, T.-Y. (2021). A likelihood-based preference ranking organization method using dual point operators for multiple criteria decision analysis in Pythagorean fuzzy uncertain contexts. Expert Systems with Applications, 176(August). Article ID 114881, 32 pp. https://doi.org/10.1016/j.eswa.2021.114881.

Chen, Y., Munir, M., Mahmood, T., Hussain, A., Zeng, S. (2021). Some generalized T-spherical and group-generalized fuzzy geometric aggregation operators with application in MADM problems. *Journal of Mathematics*, 2021(April). Article ID 5578797, 17 pp. https://doi.org/10.1155/2021/5578797.

Cuong, B.C. (2014). Picture fuzzy sets. *Journal of Computer Science and Cybernetics*, 30(4), 409–420. https://doi.org/10.15625/1813-9663/30/4/5032.

Dogan, O. (2021). Process mining technology selection with spherical fuzzy AHP and sensitivity analysis. Expert Systems with Applications, 178(September). Article ID 114999, 10 pp. https://doi.org/10.1016/j.eswa.2021.114999.

Donyatalab, Y., Seyfi-Shishavan, S.A., Farrokhizadeh, E., Gundogdu, F.K., Kahraman, C. (2020). Spherical fuzzy linear assignment method for multiple criteria group decision-making problems. *Informatica*, 31(4), 707–722. https://doi.org/10.15388/20-INFOR433.

Esangbedo, M.O., Bai, S., Mirjalili, S., Wang, Z. (2021). Evaluation of human resource information systems using grey ordinal pairwise comparison MCDM methods. *Expert Systems with Applications*, 182(November). Article ID 115151, 17 pp. https://doi.org/10.1016/j.eswa.2021.115151.

Farrokhizadeh, E., Seyfi-Shishavan, S.A., Gündoğdu, F.K., Donyatalab, Y., Kahraman, C., Seifi, S.H. (2021). A spherical fuzzy methodology integrating maximizing deviation and TOPSIS methods. *Engineering Applications of Artificial Intelligence*, 101(May). Article ID 104212, 14 pp. https://doi.org/10.1016/j.engappai.2021.104212.

Frank, D. (2014). Biodiversity, conservation biology, and rational choice. *Studies in History and Philosophy of Biological and Biomedical Sciences*, 45(1), 101–104. https://doi.org/10.1016/j.shpsc.2013.10.005.

Gao, X., Deng, Y. (2021). Generating method of Pythagorean fuzzy sets from the negation of probability. *Engineering Applications of Artificial Intelligence*, 105(October). Article ID 104403, 11 pp. https://doi.org/10.1016/j.engappai.2021.104403.

Garg, H. (2021a). CN-q-ROFS: connection number-based q-rung orthopair fuzzy set and their application to decision-making process. *International Journal of Intelligent Systems*, 36(7), 3106–3143. https://doi.org/10.1002/int.22406.

Garg, H. (2021b). Sine trigonometric operational laws and its based Pythagorean fuzzy aggregation operators for group decision-making process. *Artificial Intelligence Review*, 54(6), 4421–4447. https://doi-org.proxy.lib.cgu.edu.tw:2443/10.1007/s10462-021-10002-6.

Garg, H., Rani, D. (2021). An efficient intuitionistic fuzzy MULTIMOORA approach based on novel aggregation operators for the assessment of solid waste management techniques. *Applied Intelligence*. https://doi.org/10.1007/s10489-021-02541-w. in press.

Garg, H., Munir, M., Ullah, K., Mahmood, T., Jan, N. (2018). Algorithm for T-spherical fuzzy multi-attribute decision making based on improved interactive aggregation operators. *Symmetry*, 10(12). Article ID 670, 23 pp. https://doi.org/10.3390/sym10120670.

Garg, H., Ullah, K., Mahmood, T., Hassan, N., Jan, N. (2021). T-spherical fuzzy power aggregation operators and their applications in multi-attribute decision making. *Journal of Ambient Intelligence and Humanized Computing*, 12(10), 9067–9080. https://doi.org/10.1007/s12652-020-02600-z.

Gül, S. (2021). Extending ARAS with integration of objective attribute weighting under spherical fuzzy environment. *International Journal of Information Technology and Decision Making*, 20(3), 1011–1036. https://doi.org/10.1142/S0219622021500267.

Guleria, A., Bajaj, R.K. (2021). T-spherical fuzzy soft sets and its aggregation operators with application in decision-making. *Scientia Iranica*, 28(2), 1014–1029. https://doi.org/10.24200/SCI.2019.53027.3018.

Gündoğdu, F.K., Kahraman, C. (2019). Spherical fuzzy sets and spherical fuzzy TOPSIS method. *Journal of Intelligent & Fuzzy Systems*, 36(1), 337–352. https://doi.org/10.3233/JIFS-181401.

Hinloopen, E., Nijkamp, P. (1990). Qualitative multiple criteria choice analysis. *Quality & Quantity*, 24(1), 37–56. https://doi.org/10.1007/BF00221383.

Hinloopen, E., Nijkamp, P., Rietveld, P. (1983a). Qualitative discrete multiple criteria choice models in regional planning. *Regional Science and Urban Economics*, 13(1), 77–102. https://doi.org/10.1016/0166-0462(83)90006-6.

Hinloopen, E., Nijkamp, P., Rietveld, P. (1983b). *The REGIME Method: A New Multicriteria Technique. Essays and Surveys on Multiple Criteria Decision Making*. Springer, Berlin, Heidelberg, pp. 146–155.

Jana, C., Muhiuddin, G., Pal, M. (2021). Multi-criteria decision making approach based on SVTrN Dombi aggregation functions. *Artificial Intelligence Review*, 54(June), 3685–3723. https://doi.org/10.1007/s10462-020-09936-0.

Jin, C., Ran, Y., Wang, Z., Zhang, G. (2021). Prioritization of key quality characteristics with the three-dimensional HoQ model-based interval-valued spherical fuzzy-ORESTE method. *Engineering Applications of Artificial Intelligence*, 104(September). Article ID 104271, 18 pp. https://doi.org/10.1016/j.engappai.2021.104271.

Ju, Y., Liang, Y., Luo, C., Dong, P., Santibanez Gonzalez, E.D.R., Wang, A. (2021). T-spherical fuzzy TODIM method for multi-criteria group decision-making problem with incomplete weight information. *Soft Computing*, 25(4), 2981–3001. https://doi.org/10.1007/s00500-020-05357-x.

Kamran, M.A., Khoshsirat, M., Mikaeil, R., Nikkhoo, F. (2017). Ranking the sawability of ornamental and building stones using different MCDM methods. *Journal of Engineering Research*, 5(3), 125–149.

Karaaslan, F., Hunu, F. (2020). Type-2 single-valued neutrosophic sets and their applications in multi-criteria group decision making based on TOPSIS method. *Journal of Ambient Intelligence and Humanized Computing*, 11(10), 4113–4132. https://doi.org/10.1007/s12652-020-01686-9.

Khan, M.J., Ali, M.I., Kumam, P. (2021a). A new ranking technique for q-rung orthopair fuzzy values. *International Journal of Intelligent Systems*, 36(1), 558–592. https://doi.org/10.1002/int.22311.

Khan, Q., Gwak, J., Shahzad, M., Alam, M.K. (2021b). A novel approached based on T-spherical fuzzy Schweizer-Sklar power Heronian mean operator for evaluating water reuse applications under uncertainty. *Sustainability (Switzerland)*, 13(13). Article ID 7108, 35 pp. https://doi.org/10.3390/su13137108.

Liu, P., Ali, Z., Mahmood, T. (2021a). Novel complex T-spherical fuzzy 2-tuple linguistic Muirhead mean aggregation operators and their application to multi-attribute decision-making. *International Journal of Computational Intelligence Systems*, 14(1), 295–331. https://doi.org/10.2991/ijcis.d.201207.003.

Liu, P., Wang, D., Zhang, H., Yan, L., Li, Y., Rong, L. (2021b). Multi-attribute decision-making method based on normal T-spherical fuzzy aggregation operator. *Journal of Intelligent and Fuzzy Systems*, 40(5), 9543–9565. https://doi.org/10.3233/JIFS-202000.

Mahmood, T., Ullah, K., Khan, Q., Jan, N. (2019). An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets. *Neural Computing and Applications*, 31(11), 7041–7053. https://doi.org/10.1007/s00521-018-3521-2.

Massei, G., Rocchi, L., Paolotti, L., Greco, S., Boggia, A. (2014). Decision support systems for environmental management: a case study on wastewater from agriculture. *Journal of Environmental Management*, 146(December), 491–504. https://doi.org/10.1016/j.jenvman.2014.08.012.

Mathew, M., Chakrabortty, R.K., Ryan, M.J. (2020). A novel approach integrating AHP and TOPSIS under spherical fuzzy sets for advanced manufacturing system selection. *Engineering Applications of Artificial Intelligence*, 96(November). Article ID 103988, 13 pp. https://doi.org/10.1016/j.engappai.2020.103988.

Munir, M., Kalsoom, H., Ullah, K., Mahmood, T., Chu, Y.-M. (2020). T-spherical fuzzy Einstein hybrid aggregation operators and their applications in multi-attribute decision making problems. *Symmetry*, 12(3). Article ID 365, 24 pp. https://doi.org/10.3390/sym12030365.

Munir, M., Mahmood, T., Hussain, A. (2021). Algorithm for T-spherical fuzzy MADM based on associated immediate probability interactive geometric aggregation operators. *Artificial Intelligence Review*, 54(December), 6033–6061. https://doi.org/10.1007/s10462-021-09959-1.

Nabeeh, N.A., Abdel-Basset, M., Soliman, G. (2021). A model for evaluating green credit rating and its impact on sustainability performance. *Journal of Cleaner Production*, 280(1). Article ID 124299, 16 pp. https://doi.org/10.1016/j.jclepro.2020.124299.

Özlü, Ş., Karaaslan, F. (2021). Correlation coefficient of T-spherical type-2 hesitant fuzzy sets and their applications in clustering analysis. *Journal of Ambient Intelligence and Humanized Computing*. https://doi.org/10.1007/s12652-021-02904-8. In press.

Oztaysi, B., Onar, S.C., Kahraman, C. (2021). Waste disposal location selection by using Pythagorean fuzzy REGIME method. *Journal of Intelligent & Fuzzy Systems*. https://doi.org/10.3233/JIFS-219199. In press.

Oztaysi, B., Kahraman, C., Onar, S.C. (2022). Spherical fuzzy REGIME method waste disposal location selection. In: Kahraman, C., Cebi, S., Onar, S.C., Oztaysi, B., Tolga, A.C., Sari, I.U. (Eds.), *Intelligent and Fuzzy Techniques for Emerging Conditions and Digital Transformation. INFUS 2021*, Lecture Notes in Networks and Systems, Vol. 308. Springer, Cham. https://doi.org/10.1007/978-3-030-85577-2_84.

Pamucar, D., Yazdani, M., Obradovic, R., Kumar, A., Torres-Jiménez, M. (2020). A novel fuzzy hybrid neutrosophic decision-making approach for the resilient supplier selection problem. *International Journal of Intelligent Systems*, 35(12), 1934–1986. https://doi.org/10.1002/int.22279.

Qin, K., Wang, L. (2020). New similarity and entropy measures of single-valued neutrosophic sets with applications in multi-attribute decision making. *Soft Computing*, 24(21), 16165–16176. https://doi.org/10.1007/s00500-020-04930-8.

Şahin, R., Liu, P. (2017). Correlation coefficient of single-valued neutrosophic hesitant fuzzy sets and its applications in decision making. *Neural Computing and Applications*, 28(6), 1387–1395. https://doi.org/10.1007/s00521-015-2163-x.

Senapati, T., Yager, R.R. (2019a). Fermatean fuzzy weighted averaging/geometric operators and its application in multi-criteria decision-making methods. *Engineering Applications of Artificial Intelligence*, 85(October), 112–121. https://doi.org/10.1016/j.engappai.2019.05.012.

Senapati, T., Yager, R.R. (2019b). Some new operations over Fermatean fuzzy numbers and application of Fermatean fuzzy WPM in multiple criteria decision making. *Informatica*, 30(2), 391–412. https://doi.org/10.15388/Informatica.2019.211.

Shahzadi, G., Zafar, F., Alghamdi, M.A. (2021). Multiple-attribute decision-making using Fermatean fuzzy Hamacher interactive geometric operators. *Mathematical Problems in Engineering*, 2021(June). Article ID 5150933, 20 pp. https://doi.org/10.1155/2021/5150933.

Smarandache, F. (2005a). *A unifying field in logics: Neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability and statistics*, fourth ed. American Research Press, Rehoboth.

Smarandache, F. (2005b). Neutrosophic set–a generalization of the intuitionistic fuzzy set. *International Journal of Pure and Applied Mathematics*, 24(3), 287–297. https://ijpam.eu/contents/2005-24-3/1/1.pdf.

Smarandache, F. (2019). Neutrosophic set is a generalization of intuitionistic fuzzy set, inconsistent intuitionistic fuzzy set (picture fuzzy set, ternary fuzzy set), Pythagorean fuzzy set, spherical fuzzy set, and q-rung orthopair fuzzy set, while neutrosophication is a generalization of regret theory, grey system theory, and three-ways decision (revisited). *Journal of New Theory*, 29(December), 1–32. https://dergipark.org.tr/tr/pub/jnt/issue/51172/666629.

Stratigea, A., Grammatikogiannis, E. (2012). A multicriteria decision support framework for assessing alternative wind park locations: the case of Tanagra – Boiotia. *Regional Science Inquiry*, 4(1), 105–120.

Tsigdinos, S., Vlastos, T. (2021). Exploring ways to determine an alternative strategic road network in a metropolitan city: a multi-criteria analysis approach. *IATSS Research*, 45(1), 102–115. https://doi.org/10.1016/j.iatssr.2020.06.002.

Ullah, K., Garg, H., Mahmood, T., Jan, N., Ali, Z. (2020a). Correlation coefficients for T-spherical fuzzy sets and their applications in clustering and multi-attribute decision making. *Soft Computing*, 24(3), 1647–1659. https://doi.org/10.1007/s00500-019-03993-6.

Ullah, K., Mahmood, T., Garg, H. (2020b). Evaluation of the performance of search and rescue robots using T-spherical fuzzy Hamacher aggregation operators. *International Journal of Fuzzy Systems*, 22(2), 570–582. https://doi.org/10.1007/s40815-020-00803-2.

Ullah, K., Ali, Z., Mahmood, T., Garg, H., Chinram, R. (2021a). Methods for multi-attribute decision making, pattern recognition and clustering based on T-spherical fuzzy information measures. *Journal of Intelligent & Fuzzy Systems*. https://doi.org/10.3233/JIFS-210402. In press.

Ullah, K., Garg, H., Gul, Z., Mahmood, T., Khan, Q., Ali, Z. (2021b). Interval valued T-spherical fuzzy information aggregation based on Dombi t-norm and Dombi t-conorm for multi-attribute decision making problems. *Symmetry*, 13(6). Article ID 1053, 26 pp. https://doi.org/10.3390/sym13061053.

Wang, J.-C., Chen, T.-Y. (2021). A T-spherical fuzzy ELECTRE approach for multiple criteria assessment problem from a comparative perspective of score functions. *Journal of Intelligent & Fuzzy Systems*, 41(2), 3751–3770. https://doi.org/10.3233/JIFS-211431.

Wątróbski, J., Jankowski, J., Ziemba, P., Karczmarczyk, A., Zioło, M. (2019). Generalised framework for multi-criteria method selection. *Omega (United Kingdom)*, 86(July), 107–124. https://doi.org/10.1016/j.omega.2018.07.004.

Xu, Z.S. (2005). An overview of methods for determining OWA weights. *International Journal of Intelligent Systems*, 20(8), 843–865. https://doi.org/10.1002/int.20097.

Yager, R.R. (2013). Pythagorean fuzzy subsets. In: *Proceedings of the 2013 Joint IFSA World Congress and NAFIPS Annual Meeting*, Edmonton, Canada, June 24–28, 2013, pp. 57–61. https://doi.org/10.1109/IFSA-NAFIPS.2013.6608375.

Yager, R.R. (2017). Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems*, 25(5), 1222–1230. https://doi.org/10.1109/TFUZZ.2016.2604005.

Yager, R.R., Abbasov, A.M. (2013). Pythagorean membership grades, complex numbers, and decision making. *International Journal of Intelligent Systems*, 28(5), 436–452. https://doi.org/10.1002/int.21584.

Zeng, S., Garg, H., Munir, M., Mahmood, T., Hussain, A. (2019). A multi-attribute decision making process with immediate probabilistic interactive averaging aggregation operators of T-spherical fuzzy sets and its application in the selection of solar cells. *Energies*, 12(23). Article ID 4436, 26 pp. https://doi.org/10.3390/en12234436.

Zeng, S., Munir, M., Mahmood, T., Naeem, M. (2020). Some T-spherical fuzzy Einstein interactive aggregation operators and their application to selection of photovoltaic cells. *Mathematical Problems in Engineering*, 2020(June). Article ID 1904362, 16 pp. https://doi.org/10.1155/2020/1904362.

T.-Y. Chen is currently a professor in the Department of Industrial and Business Management and the Graduate Institute of Management at Chang Gung University in Taiwan. She received a BS degree in transportation engineering and management, an MS degree in civil engineering, and a PhD degree in traffic and transportation from Chiao Tung University in Taiwan. She has been a visiting professor in the Institute of Information Science at Academia Sinica and in the Department of Information Management at National Chi Nan University. She has been an Adjunct Research Fellow in the Department of Nursing and in the Division of Cerebrovascular Disease of the Department of Neurology at Linkou Chang Gung Memorial Hospital. Her current research interests include multiple criteria decision analysis, fuzzy decision making in modelling, and intelligent decision support for management. She is an Honorary Member of the Phi Tau Phi Scholastic Honor Society of Taiwan.