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A Novel Extended Fuzzy Pivot Pairwise Relative Criteria Importance Assessment (MF PIPRECIA-E) Method

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Abstract. This study proposes a Modified Fuzzy Extended Pivot Pairwise Relative Criteria Importance Assessment (MF PIPRECIA-E) method for determining criteria weights under uncertainty. The proposed extension enables the upper bounds of fuzzy numbers obtained during calculation to remain below, equal to, or slightly above 1. In addition, this study also critically examines the procedure of direct and inverse assignment of the relative importance of the criteria, integrated in the PIPRECIA-E method, with the aim of pointing out some important characteristics of this method, which have not been sufficiently explored in previous research. The proposed approach also addresses consistency assessment and group decision-making. Therefore, the main contribution of this study is development of a modified fuzzy PIPRECIA-E method that provides a more consistent procedure for determining weights of the criteria under uncertainty. Accordingly, proposed approach

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incorporates both direct and inverse criteria importance assessment, extending the applicability of the PIPRECIA-E method for solving various multiple-criteria decision-making (MCDM) problems.

Key words: criteria weights, PIPRECIA, PIPRECIA-E, MF PIPRECIA-E, fuzzy sets, triangular fuzzy numbers.

1. Introduction

In addition to determining performances of alternatives in relation to the selected set of evaluation criteria, determining criteria weight also has a significant impact on the evaluation of the alternatives by applying Multiple Criteria Decision Making (MCDM) methods (Bedirhanoğlu, 2026; Nejkovic *et al.*, 2022; Chun-Yueh, 2022). For this reason, numerous subjective and objective methods for determining weights of criteria have been proposed so far (Mert and Aytekin, 2025; Korucuk and Aytekin, 2025).

The Pivot Pairwise Relative Criteria Importance Assessment (PIPRECIA) method (Stanujkic *et al.*, 2017) was originally developed and proposed as an adaptation of the Step-Wise Weight Assessment Ratio Analysis (SWARA) method (Keršuliene *et al.*, 2010). Recent research has also focused on improving fuzzy extensions of the SWARA method, including the development of an adapted bounded fuzzy SWARA extension in which values are constrained within an appropriate interval (Stanujkic *et al.*, 2026).

The main difference, that is, the advantage of the PIPRECIA method compared to the SWARA method is that it does not require consensus regarding the expected significance of the criteria. In other words, the PIPRECIA method has a much wider scope of use than the SWARA method.

The PIPRECIA method has been used to resolve various decision-making problems, demonstrating its efficiency and usefulness. Hereinafter, a few examples illustrating its applicability across various fields are presented. The PIPRECIA-S enabled the assessment of artificial intelligence applications in higher education (Ilić *et al.*, 2026), while the PIPRECIA-E method was used to assess the gender equality criteria in the field of logistics (Gurturk, 2025). Marinović *et al.* (2025) applied the PIPRECIA method to select the optimal operating system suitable for an educational environment. The personnel selection problem was successfully resolved using PIPRECIA-S in combination with the CRITIC and WISP methods (Stanujkić *et al.*, 2025). Employee job satisfaction was assessed using the PIPRECIA-S (Sarbat, 2024). The applied leadership model during the reforms in Serbia's public sector was assessed using the PIPRECIA-S method (Janošik *et al.*, 2023). The significance of the selected criteria influencing the decision to select the hotel construction project was defined using the PIPRECIA method (Popovic *et al.*, 2022). The PIPRECIA method helped define the importance of comparative indicators for sustainable tourism (Janovac *et al.*, 2023). The significance of the criteria was successfully determined in the case of bus selection (Ozdagoglu *et al.*, 2022). Popovic *et al.* (2021) identified the key factors influencing tourism development in Serbia using the PIPRECIA-E. The potential of the methods from the PIPRECIA family was further expanded by creating useful extensions using fuzzy, grey, and neutrosophic numbers. Until now, the authors have proposed single-valued Pythagorean PIPRECIA (Turan and Büyükselçuk, 2026), Z-cloud rough

fuzzy numbers PIPRECIA (Alnoor *et al.*, 2025), single-valued neutrosophic PIPRECIA (Popovic *et al.*, 2025), interval fuzzy rough PIPRECIA (Song *et al.*, 2025; Radović *et al.*, 2024), Trapezoidal interval type-2 fuzzy (Xu *et al.*, 2023), plithogenic PIPRECIA (Ulutaş *et al.*, 2021), grey PIRECIA (Stanujkić *et al.*, 2021; Ulutaş *et al.*, 2020), fuzzy PIPRECIA (Stević *et al.*, 2018).

The proposed fuzzy extensions of the PIPRECIA method are based on the application of triangular fuzzy numbers, which enable the parameters that define the boundaries of the triangle, i.e. the left boundary, mode and right boundary, to have values greater than 1. That is why the values of the parameters obtained by applying the computational procedure of these fuzzy extensions of the PIPRECIA method are, in some cases, greater than 1.

Therefore, this article proposes a modification of the fuzzy PIPRECIA method, in which the values of the parameters during the calculation belong to the interval [0, 1]. The proposed modification gives the same results as the existing fuzzy extensions of the PIPRECIA method, with the fact that it is not proposed to replace the existing fuzzy extensions of the method, but to serve as a basis for proposing Intuitionistic, Pythagorean, Spherical or Neutrosophic extensions of the PIPRECIA method.

To present the proposed modification of the fuzzy PIPRECIA method (MF PIPRECIA), the remaining part of this article is organized as follows: In Section 2, the PIPRECIA method and its fuzzy extension is presented and the Simplified PIPRECIA method is proposed, while in Section 3 MF PIPRECIA is proposed. In Section 4, the usage of the MF PIPRECIA method for determining the criteria weights is presented. Finally, conclusions are given.

2. Preliminaries

The procedure of the PIPRECIA method, as well as some basic elements of fuzzy logic used for developing a new fuzzy extension of the PIPRECIA method, are presented.

2.1. *PIPRECIA and Inverse PIPRECIA Method*

The PIPRECIA method integrates two procedures for determining criteria weight, the first known as the PIPRECIA method and the second known as the inverse PIPRECIA method, which are together integrated into the Extended PIPRECIA (PIPRECIA-E) method (Stanujkic *et al.*, 2017).

In the first approach, each criterion, except the first one, the significance of each criterion is compared with the significance of the previous criterion, whereby the comparison starts from the second criterion in the list of criteria, and ends with the last one in the list of criteria. This weighting procedure is often used independently and is known as the PIPRECIA method.

The second procedure for determining criteria weights is somewhat like the procedure used in the PIPRECIA method. However, in the inverse procedure (PIPRECIA-I) method, the comparison of the relative criteria importance begins from the $n - 1$ criterion and

ends with the first criterion in the list of criteria. The computational procedure for determining criteria weights using the PIPRECIA-I procedure is very similar to the procedure of the PIPRECIA procedure, but this pairwise comparison procedure can sometimes be much more complex for the respondent, which is why there are often certain deviations in the criteria weights obtained using the PIPRECIA and PIPRECIA-I procedures. However, combining the above two approaches can be used to more realistically determine the significance of the criteria.

2.1.1. *PIPRECIA Method*

The procedure of determining criteria weights using the PIPRECIA method is precisely explained in Stanujkic *et al.* (2017), but for the consistency of the presentation of the newly proposed fuzzy modification of the PIPRECIA method, this process is presented again in Appendix A – The PIPRECIA method, while an example of applying the PIPRECIA method is presented in Appendix B – Criteria weights determination using the PIPRECIA method.

2.1.2. *PIPRECIA-I Method*

The procedure for determining criteria weights using the PIPRECIA-I method is also described in detail in Stanujkic *et al.* (2017). As in the previous case, due to the consistency of the further presentation, this procedure is presented in Appendix C – The PIPRECIA-I method, while an example of determining the importance of criteria using the mentioned approach is shown in Appendix D – Criteria weights determination using the PIPRECIA-I method.

2.1.3. *Checking the Consistency of the Comparisons*

In the original version of the PIPRECIA-E method, Spearman's rank correlation coefficient was used to determine the consistency of respondents' responses, which is their attitudes regarding the relative importance of the criteria.

However, in this adaptation of the PIPRECIA-E method, the use of normalized Hamming distance between the criteria weights obtained using the PIPRECIA and PIPRECIA-I methods is considered for determining consistency of the comparisons.

Let w'_j and w''_j, $j = 1, \ldots, n$, be criteria weights obtained applying the PIPRECIA and PIPRECIA-I methods. Then, consistency of the comparisons c_c can be calculated as follows:

$$c_c = \frac{1}{1 + \frac{1}{n}\sum_{j=1}^{n} \lfloor w'_j - w''_j \rfloor}. \tag{1}$$

However, in real cases of using the PIPRECIA-E method, the values of σ are greater than 0, and decision makers should define an acceptable level of the value of σ, depending on the problem being solved.

2.1.4. *Determination of the Resulting Criteria Weights*

In the case of applying the direct and inverse procedure of determining the weights of the PIPRECIA-E method, two weight vectors, w'_j and w''_j, are formed, and it is realistic

to expect that there is a certain difference in the weights of the criteria obtained by applying the direct and inverse procedure. Regarding the original version of the PIPRECIA method (Stanujkic *et al.*, 2017), or more precisely the PIPRECIA-E method, the following equation was proposed to determine the resulting criteria weights:

$$w_j = \frac{w'_j + w''_j}{2}. \tag{2}$$

However, taking into account that the examples of the real application of the PIPRECIA-E method indicated that the application of the inverse procedure for evaluating the importance of the criteria can be significantly more complex for the respondents, in this approach the following, significantly more flexible approach for determining the resulting weights of the criteria is proposed:

$$w_j = \alpha w'_j + (1 - \alpha) w''_j, \tag{3}$$

where α is a coefficient and $\alpha \epsilon [0, 1]$.

2.2. *Some Basic Elements of Fuzzy Sets and Triangular Fuzzy Numbers*

The fuzzy set (FS) theory was proposed by Zadeh (1965), as an extension of a classical set theory. In contrast to the classical set theory, FS theory allows for partial membership to a set.

Definition 1 (Ejegwa, 2019). A fuzzy subset $\tilde{A}$ of a non-empty set X is defined as follows:

$$\tilde{A} = \left\{ \left\langle x, \mu_{\tilde{A}}(x) \right\rangle \middle| \, x \in X \right\}, \tag{4}$$

where $\mu_{\tilde{A}}(x) : X \to [0, 1]$ is the membership function of FS $\tilde{A}$, $\mu_{\tilde{A}}(x) \in [0, 1]$ is the degree of membership of the element x to the $\tilde{A}$.

Membership functions can have different shapes such as bell-shaped, Gaussian, trapezoidal, and triangular.

Definition 2 (Cheng, 2004). Triangular fuzzy number (TFN) $\tilde{A}$ can be represented as a triplet $\tilde{A} = (l, m, u)$ where membership function is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} (x - l)/(m - l), & l \leqslant x < m, \\ 1, & x = m, \\ (u - x)/(u - m), & m < x \leqslant u, \\ 0, & \text{otherwise}, \end{cases} \tag{5}$$

where l, m, and u denote left endpoint, mode, and right endpoint, respectively, of TFN $\tilde{A}$, and $l < m < u$.

Definition 3 (Chen and Hwang, 1992). Let $\tilde{A} = (a_l, a_m, a_u)$ and $\tilde{B} = (b_l, b_m, b_u)$ be two positive TFNs, and k is a positive crisp number. The arithmetic operations of these TFNs, necessary for the development of the fuzzy extension of the PIPRECIA method, are as follows:

$$\tilde{A} \oplus \tilde{B} = (a_l + b_l, a_m + b_m, a_u + b_u), \tag{6}$$

$$\tilde{A} \oslash \tilde{B} = \left(\frac{a_l}{b_u}, \frac{a_m}{b_m}, \frac{a_u}{b_l}\right), \tag{7}$$

$$\tilde{A} \cdot k = (a_l \cdot k, a_m \cdot k, a_u \cdot k), \tag{8}$$

$$\frac{\tilde{A}}{k} = \left(a_l \cdot \frac{1}{k}, a_m \cdot \frac{1}{k}, a_u \cdot \frac{1}{k}\right). \tag{9}$$

Definition 4. Let $F_i = (l_i, m_i, u_i)$ be a collection of n fuzzy numbers. The fuzzy weighted arithmetic (FWA) operator of F_i is as follows:

$$FWA_{(F_j)} = \sum_{j=1}^{n} w_j F_j = \left(\sum_{j=1}^{n} w_j l_j, \sum_{j=1}^{n} w_j m_j, \sum_{j=1}^{n} w_j u_j\right), \tag{10}$$

where: w_j denotes the weight of element j of the collection A_j, $w_j \in [0, 1]$, and $\sum_{j=1}^{n} w_j = 1$.

2.3. *Modified PIPRECIA-E Method*

This subsection presents an adaptation of the PIPRECIA-E method, which enables all values of variables used in the calculation procedure of the PIPRECIA-E method to be kept in the interval [0, 1].

2.3.1. *Computational Procedure of the Modified PIPRECIA Method*

The computational procedure of the modified PIPRECIA (M PIPRECIA) method for determining criteria weights of n criteria can be presented through the following steps:

Step 1. Starting from the second criterion, set the value of s_j for each criterion as follows:

$$\begin{cases} s_j \in (0.5, 0.95], & \text{when } C_j \succ C_{j-1}, \\ s_j = 0.5, & \text{when } C_j = C_{j-1}, \\ s_j \in [0.05, 0.5), & \text{when } C_j \prec C_{j-1}, \end{cases} \tag{11}$$

where $j \neq 1$.

Step 2. Transform the value of s_j as follows:

$$sk_j = \begin{cases} 0.5, & \text{when } j = 1, \\ 1 - s_j, & \text{when } j \neq 1, \end{cases} \tag{12}$$

$$sq_j = \begin{cases} 0.5, & \text{when } j = 1, \\ \frac{sq_{j-1}}{2 \cdot sk_j}, & \text{when } j \neq 1, \end{cases} \tag{13}$$

$$sn_j = \frac{1}{\max_j sq_j} sq_j. \tag{14}$$

Step 3. Determine the relative weights of the evaluation criteria as follows:

$$w_j = \frac{sn_j}{\sum_{k=1}^{n} sn_k}. \tag{15}$$

The modified PIPRECIA method enables the values of the variables used in the calculation to belong to interval [0,1], whereby this modification provides the same criteria weights as the original version of the PIPRECIA method. A comparison of the results of the PIPRECIA and M PIPRECIA methods is presented in Appendix E – Criteria weights determination using the modified PIPRECIA method.

2.3.2. *Computational Procedure of the Modified Inverse PIPRECIA Method*
The computational procedure of the modified inverse PIPRECIA (MI PIPRECIA) method for determining criteria weights of n criteria can be presented through the following steps:

Step 1. Starting from the second to last criterion, set the value of s_j for each criterion as follows:

$$\begin{cases} s_j \in (0.5, 0.95], & \text{when } C_j \succ C_{j+1}, \\ s_j = 0.5, & \text{when } C_j = C_{j+1}, \\ s_j \in [0.05, 0.5), & \text{when } C_j \prec C_{j+1}, \end{cases} \tag{16}$$

where $j \neq n$.

Step 2. Transform the value of s_j as follows:

$$sk_j = \begin{cases} 0.5, & \text{when } j = n, \\ 1 - s_j, & \text{when } j \neq n, \end{cases} \tag{17}$$

$$sq_j = \begin{cases} 0.5, & \text{when } j = n, \\ \frac{sq_{j-1}}{2 \cdot sk_j}, & \text{when } j \neq n, \end{cases} \tag{18}$$

$$sn_j = \frac{1}{\max_j sq_j} sq_j. \tag{19}$$

Step 3. Determine the relative weights of the evaluation criteria as follows:

$$w_j = \frac{sn_j}{\sum_{k=1}^{n} sn_k}. \tag{20}$$

As in the case of the modified PIPRECIA method, the modified inverse PIPRECIA method enables the values of the variables used in the calculation belong to interval [0, 1], whereby this modification provides the same criterion weights as the original inverse version of the PIPRECIA method. A comparison of the results obtained using the inverse PIPRECIA and IM PIPRECIA methods is presented in Appendix F – Criteria weights determination using the modified inverse PIPRECIA method.

3. A Novel Modified Fuzzy PIPRECIA-E Method

A novel modified fuzzy PIPRECIA-E method integrates modified PIPRECIA-E method and fuzzy logic concept, or more precisely, usage of triangular fuzzy numbers, for determining criteria weights, which provides possibility for more realistic determining criteria weights in cases when decision-making problem is related with predictions and uncertainties. As well as the above presented variants of the PIPRECIA method, this variant also integrates a direct and inverse approach for determining criteria weights.

3.1. *A Novel Fuzzy PIPRECIA Method*

The computational procedure of the modified fuzzy PIPRECIA (MF PIPRECIA) method for determining criteria weights of n criteria can be presented through the following steps:

Step 1. Assigning the relative significance s_j of each criterion, excepting the first criterion, using triangular fuzzy numbers, as follows:

$$\tilde{s}_j(l_j, m_j, u_j) = \begin{cases} s_j = (\geqslant 0.5, \geqslant 0.5 \wedge \leqslant 0.95, \leqslant 0.95), & \text{when } C_j \succ C_{j-1}, \\ s_j = (0.5, 0.5, 0.5), & \text{when } C_j = C_{j-1}, \\ s_j \in (\geqslant 0.05, \geqslant 0.05 \wedge \leqslant 0.5, \leqslant 0.5), & \text{when } C_j \prec C_{j-1}, \end{cases} \tag{21}$$

where $j \neq 1$.

Step 2. Transform the value of $\tilde{s}_j$ as follows:

$$\tilde{sk}_j(l_j, m_j, u_j) = \begin{cases} (0.5, 0.5, 0.5), & \text{when } j = 1, \\ \tilde{1} - \tilde{s}_j, & \text{when } j \neq 1, \end{cases} \tag{22}$$

$$\tilde{sq}_j(l_j, m_j, u_j) = \begin{cases} (0.5, 0.5, 0.5), & \text{when } j = 1, \\ \frac{\tilde{sq}_{j-1}}{2 \cdot \tilde{sk}_j}, & \text{when } j \neq 1, \end{cases} \tag{23}$$

$$\tilde{sn}_j(l_j, m_j, u_j) = \frac{1}{\max_j u_j} \tilde{sq}_j(l_j, m_j, u_j). \tag{24}$$

Step 3. Determine the relative weights of the evaluation criteria as follows:

$$\tilde{w}_j(l_j, m_j, u_u) = \frac{\tilde{sn}_j}{\sum_{k=1}^{n} \tilde{sn}_k}. \tag{25}$$

The weights of the criteria obtained by applying the previously presented procedure are fuzzy or triangular fuzzy numbers. Depending on the further calculation procedure, these weights can be used in their existing form, or if necessary, they can be transformed into crisp values, which is a more common approach.

The simplest way to transform fuzzy weight $\tilde{w}_j(l_j, m_j, u_j)$ into crisp values w_j is as follows:

$$w_j^c = \frac{l_j + m_j + u_j}{3}. \tag{26}$$

However, Eq. (25) does not ensure the fulfillment of the condition $\sum_{j=1}^{n} w_j^c = 1$, which is why it is necessary to determine the final weights as follows:

$$w_j = \frac{w_j^c}{\sum_{k=1}^{n} w_k^c}. \tag{27}$$

In addition to the previously presented approach for transforming fuzzy to crisp weights, the following transformation approaches, which enable giving different weights to the left and right boundaries of the triangular fuzzy number, or the mode of the fuzzy numbers, can also be used:

$$w_j^c = \frac{\alpha \cdot l_j + m_j + (1-\alpha) \cdot u_j}{2}, \tag{28}$$

$$w_j^c = \frac{\alpha \cdot l_j + \beta \cdot m_j + (1-\alpha) \cdot u_j}{3}, \tag{29}$$

where coefficient $\alpha \in [0, 1]$, and coefficient $\beta \in [0, 2]$. By varying the values of coefficients α and β, decision-makers can change the influence, of the left and right bounds, and/or mode, of triangular fuzzy numbers, and in this way consider different analyses, or scenarios.

As in the case of using Eq. (26), it is also necessary to apply Eq. (27) in these cases to ensure that the sum of criteria weights is equal to 1.

3.2. *A Novel Inverse Fuzzy PIPRECIA Method*

The computational procedure of the modified inverse fuzzy PIPRECIA (MFI PIPRECIA) method for determining criteria weights of n criteria can be represented through the following steps:

Step 1. Assigning the relative significance $\tilde{s}_j$ of each criterion, excepting the first criterion, using triangular fuzzy numbers, as follows:

$$\tilde{s}_j(l_j, m_j, u_j) = \begin{cases} s_j = (\geqslant 0.5, \geqslant 0.5 \wedge \leqslant 0.95, \leqslant 0.95), & \text{when } C_j \succ C_{j+1}, \\ s_j = (0.5, 0.5, 0.5), & \text{when } C_j = C_{j+1}, \\ s_j \in (\geqslant 0.05, \geqslant 0.05 \wedge \leqslant 0.5, \leqslant 0.5), & \text{when } C_j \prec C_{j+1}, \end{cases} \tag{30}$$

where $j \neq n$.

Step 2. Transform the value of s_j as follows:

$$\tilde{sk}_j(l_j, m_j, u_j) = \begin{cases} (0.5, 0.5, 0.5), & \text{when } j = n, \\ \tilde{1} - \tilde{s}_j, & \text{when } j \neq n, \end{cases} \tag{31}$$

$$\tilde{sq}_j(l_j, m_j, u_j) = \begin{cases} (0.5, 0.5, 0.5), & \text{when } j = n, \\ \frac{\tilde{sq}_{j+1}}{2 \cdot \tilde{sk}_j}, & \text{when } j \neq n, \end{cases} \tag{32}$$

$$\tilde{sn}_j(l_j, m_j, u_j) = \frac{1}{\max_j u_j} \tilde{sq}_j(l_j, m_j, u_j). \tag{33}$$

Step 3. Determine the relative weights of the evaluation criteria as follows:

$$\tilde{w}_j(l_j, m_j, u_u) = \frac{\tilde{sn}_j}{\sum_{k=1}^{n} \tilde{sn}_k}. \tag{34}$$

3.3. *Checking the Consistency of the Comparisons*

As a result of using the MF PIPRECIA-E method, two fuzzy criterion weights are actually obtained, which are realistically different to some extent, especially if we consider that the inverse criteria evaluation procedure can be significantly more difficult for the examinee. In the case of transforming fuzzy weights into crisp values, consistency and weights can be determined as in the PIPRECIA-E method, that is using Eq. (1).

However, the weights obtained by the MF PIPRECIA-E method do not always have to be transformed into crisp values, because they can still be used in a fuzzy form for further calculations. In this case, the consistency of the performed pairwise comparison in the direct and indirect form can be determined as follows:

$$c_c = \frac{1}{1 + \frac{1}{3n} \sum_{j=1}^{n} (\lfloor l'_j - l''_j \rfloor + \lfloor m'_j - m''_j \rfloor + \lfloor u'_j - u''_j \rfloor)}. \tag{35}$$

3.4. *Determination of the Resulting Criteria Weights*

In the case of applying the direct and inverse procedure of determining the weights of the fuzzy PIPRECIA-E method, two fuzzy weight vectors, w'_j and w''_j, are formed.

In the case of further application of the crisp weight criteria, the fuzzy weights w'_j and w''_j can be transformed into crisp weights, and the resulting weight can be determined by applying Eq. (2) and Eq. (3). However, in the case of further calculations based on the application of fuzzy criteria weights, the resulting fuzzy weights of the criteria can be determined in the simplest way as follows:

$$\tilde{w}_j(l_j, m_j, u_u) = \begin{cases} \frac{l'_j + l''_j}{2}, \\ \frac{m'_j + m''_j}{2}, \\ \frac{u'_j + u''_j}{2}. \end{cases} \tag{36}$$

Table 1
Fuzzy linguistic scale proposed for use with MF PIPRECIA-E method.

Linguistic variable		TFN I			TFN II		
	Abbreviation	*l*	*m*	*u*	*l*	*m*	*u*
Significantly Greater Importance	SGI	0.85	0.95	0.95	0.925	0.95	0.975
Much Greater Importance	MuGI	0.75	0.85	0.95	0.825	0.85	0.875
Greater Importance	GI	0.65	0.75	0.85	0.725	0.75	0.775
Moderately Greater Importance	MoGI	0.55	0.65	0.75	0.625	0.65	0.675
Slightly Greater importance	SGI	0.50	0.50	0.55	0.475	0.50	0.525
The Same Importance	SI	0.50	0.50	0.50	0.500	0.50	0.500
Slightly Less Importance	SLI	0.45	0.50	0.50	0.475	0.50	0.525
Moderately Less Importance	MoLI	0.25	0.35	0.45	0.325	0.35	0.375
Less Importance	LI	0.15	0.25	0.35	0.225	0.25	0.275
Much Less Importance	MuLI	0.05	0.15	0.25	0.125	0.15	0.175
Significantly Less Importance	SLI	0.05	0.05	0.15	0.025	0.05	0.075

Source: Authors

Certainly, in the case of the need for carrying out additional analyses, the resulting fuzzy weights can be determined by applying significantly more complex approaches, which enable giving different meanings to the left and right boundaries of the fuzzy number, i.e. the mode of the fuzzy number.

3.5. *Fuzzy Linguistic Scales*

The use of triangular fuzzy numbers, or more precisely setting values of three parameters that define triangular fuzzy numbers, i.e. lower bound, mode, and upper bound, for expressing preferences about relative importance of criteria, can be too complex for less experienced respondents, that is, respondents who are not familiar with the use of fuzzy numbers.

For this reason, many fuzzy extensions of MCDM methods also suggest the use of appropriate linguistic variables, such as linguistic scales, to express respondents' attitudes. However, the precise expression of respondents' preferences can be limited using linguistic variables to some extent. For this reason, in the proposed fuzzy extension of PIPRECIA-E method usage of a linguistic scale whose linguistic variables can be transformed into one of the two offered fuzzy numbers, with different spreads, as shown in Table 1 and Table 2, is also proposed.

Depending on their preferences, respondents can choose one of the two offered values of triangular fuzzy numbers, and if necessary, they can also modify the proposed values.

3.6. *A Novel Modified Fuzzy PIPRECIA-E Method in a Group Decision-Making Environment*

In the case of solving multiple-criteria decision-making problems, i.e., determining the criteria, using a group decision-making approach enables problem-solving in two basic ways:

Table 2
Spreads of fuzzy linguistic scale proposed for using with MF PIPRECIA-E method.

Linguistic variable	Abbrevation	TFN I	TFN II
Significantly Greater Importance	SGI	0.10	0.05
Much Greater Importance	MuGI	0.20	0.05
Greater Importance	GI	0.20	0.05
Moderately Greater Importance	MoGI	0.20	0.05
Slightly Greater importance	SGI	0.05	0.05
The Same Importance	SI	0.00	0.00
Slightly Less Importance	SLI	0.05	0.05
Moderately Less Importance	MoLI	0.20	0.05
Less Importance	LI	0.20	0.05
Much Less Importance	MuLI	0.20	0.05
Significantly Less Importance	SLI	0.10	0.05

Source: Authors

– The first, and certainly simpler, way is to calculate the crisp values of criteria weights for each respondent and then determine the resulting group weight of the criteria.
– The second, and significantly more complete, approach is based on calculations that use group decision matrices, which were formed with the aim of fully utilizing the advantages that triangular fuzzy numbers offer. In this case, two group decision-making matrices are formed, one for the direct and the other for the indirect procedure of evaluating the significance of the criteria.

In this approach, emphasis is placed on the calculation procedure based on the application of the group decision matrix, which is formed by applying the FWA operator, that is, applying Eq. (11). After the formation of the group decision-making matrix, the further calculation procedure is like the procedure carried out in the case of calculation based on the attitudes of single respondent.

As the weakness of this approach, the difficulty of determining the consistency of the pairwise comparisons can be mentioned. In the case of calculations made based on attitudes of each respondent separately, the consistency of the comparisons can be easily checked, and respondents with inconsistent attitudes can be excluded from further calculation, or they may be asked to re-evaluate comparisons.

However, even in the case of applying group approach for determining criteria weights, the consistency of the ”group comparisons” can be determined based on the weights of the criteria obtained by the using direct and indirect comparisons, that is using Eq. (35).

4. Numerical Illustrations

To verify, as well as to emphasize the specificity of the proposed approach, two numerical illustrations are considered in this section:

– the first numerical illustration presents use of the MF PIPRECIA-E for determining criteria weights based on attitudes of one respondent, and

– the second numerical illustration also presents use of the MF PIPRECIA-E method in a group decision-making environment, also for determining criteria weights.

4.1. *The First Numerical Illustration*

As mentioned previously, the first numerical illustration presents the use of the proposed approach for determining criteria weights for evaluating customer satisfaction in traditional Serbian restaurants, based on the following criteria (Stanujkic *et al.*, 2019):

– The building's interior and hospitable atmosphere (C_1),
– A courteous and friendly staff (C_2),
– The assortment of traditional foods and beverages (C_3),
– The quality and taste of food and drinks, as well as the way it's served (C_4),
– The quality of services provided is worth the reasonable price (C_5),
– Other (C_6).

The attitudes of respondents obtained by applying the direct procedure for comprising the relative importance of the MF PIPRECIA method are shown in Table 3, while the details of the calculation are presented in Table 4.

The previously presented example of calculating criteria weights presents the rarer case of calculations in which the upper limits of the triangular fuzzy numbers are slightly greater than 1.

Table 3
Attitudes of respondent obtained using MF PIPRECIA.

Criterion	Linguistic variable	s_j		
		l	m	u
C_1				
C_2	MuGI	0.75	0.85	0.95
C_3	SGI	0.50	0.50	0.55
C_4	SGI	0.50	0.50	0.55
C_5	SLI	0.45	0.50	0.5
C_6	MoLI	0.25	0.35	0.45

Table 4
Calculation details and fuzzy criteria weights obtained using MF PIPRECIA.

Criterion	$\tilde{sk}_j$			$\tilde{sq}_j$			$\tilde{sn}_j$			$\tilde{w}_j$		
	l	m	u	l	m	u	l	m	u	l	m	u
C_1	0.5	0.5	0.5	0.5	0.5	0.5	0.017	0.017	0.017	0.017	0.059	0.100
C_2	0.05	0.15	0.25	1.00	1.67	5.00	0.034	0.057	0.172	0.034	0.197	0.997
C_3	0.45	0.50	0.50	1.00	1.67	5.56	0.034	0.057	0.191	0.034	0.197	1.108
C_4	0.45	0.50	0.50	1.00	1.67	6.17	0.034	0.057	0.213	0.034	0.197	1.231
C_5	0.50	0.50	0.55	0.91	1.67	6.17	0.031	0.057	0.213	0.031	0.197	1.231
C_6	0.55	0.65	0.75	0.61	1.28	5.61	0.021	0.044	0.193	0.021	0.152	1.119
						29.01	0.173	0.291	1.000			

Table 5
Fuzzy and crisp values of criteria weights.

Criterion	$\tilde{w}_j$			w_j^c	w_j
	l	m	u		
C_1	0.017	0.059	0.100	0.059	0.025
C_2	0.034	0.197	0.997	0.410	0.177
C_3	0.034	0.197	1.108	0.446	0.193
C_4	0.034	0.197	1.231	0.488	0.210
C_5	0.031	0.197	1.231	0.486	0.210
C_6	0.021	0.152	1.119	0.431	0.186
				2.319	1.000

Table 6
Transformation of fuzzy into crisp criteria weights using Eq. (27).

Criterion	$\tilde{w}_j$			w_j^c	w_j	w_j^c	w_j	w_j^c	w_j
	l	m	u	$\alpha = 0$		$\alpha = 0.5$		$\alpha = 1$	
C_1	0.017	0.059	0.100	0.079	0.023	0.059	0.030	0.038	0.065
C_2	0.034	0.197	0.997	0.597	0.176	0.356	0.179	0.116	0.198
C_3	0.034	0.197	1.108	0.653	0.192	0.384	0.193	0.116	0.198
C_4	0.034	0.197	1.231	0.714	0.210	0.415	0.209	0.116	0.198
C_5	0.031	0.197	1.231	0.714	0.210	0.414	0.208	0.114	0.195
C_6	0.021	0.152	1.119	0.635	0.187	0.361	0.181	0.086	0.147
				3.393	1.000	1.989	1.000	0.586	1.000

Table 7
Transformation of fuzzy into crisp criteria weights using Eq. (28).

Criterion	$\tilde{w}_j$			w_j^c	w_j	w_j^c	w_j	w_j^c	w_j
	l	m	u	$\beta = 0$		$\beta = 0.5$		$\beta = 1$	
C_1	0.017	0.059	0.100	0.019	0.019	0.039	0.037	0.059	0.054
C_2	0.034	0.197	0.997	0.173	0.166	0.174	0.164	0.176	0.162
C_3	0.034	0.197	1.108	0.202	0.193	0.202	0.189	0.202	0.186
C_4	0.034	0.197	1.231	0.224	0.214	0.224	0.210	0.224	0.206
C_5	0.031	0.197	1.231	0.225	0.215	0.225	0.211	0.225	0.207
C_6	0.021	0.152	1.119	0.201	0.193	0.201	0.189	0.201	0.185
				1.045	1.000	1.066	1.000	1.087	1.000

The values in columns $\tilde{sk}_j$, $\tilde{sq}_j$, $\tilde{sn}_j$ and $\tilde{w}_j$ were calculated using Eqs. (21) to (24). Crisp values of criteria weights obtained using Eqs. (25) and (26) are shown in Table 5.

A similar process of transforming fuzzy to crisp criterion weights could be done by applying Eqs. (27) or (28). The results of transforming fuzzy into crisp criteria weights using Eq. (27), and different values of coefficient α, is presented in Table 6.

The results of transforming fuzzy into crisp criteria weights applying Eq. (28) are presented in Table 7. To ensure a simpler presentation, in this case, only the values of coefficient β were varied, while the values of coefficient α were constant, and it was 0.5.

From the previous two tables, the application of Eqs. (27) and (28) has a certain influence on the criteria weights, which can enable decision makers to explore different sce-

Table 8
Attitudes of respondent obtained applying MFI PIPRECIA.

Criterion	Linguistic variable	$\tilde{s}_j$		
		l	m	u
C_1	MuLI	0.05	0.15	0.25
C_2	MoLI	0.25	0.35	0.45
C_3	SGI	0.50	0.50	0.55
C_4	SGI	0.50	0.50	0.55
C_5	MoGI	0.55	0.65	0.75
C_6				

Table 9
Calculation details and fuzzy criteria weights obtained using MFI PIPRECIA.

Criterion	$\tilde{sk}_j$			$\tilde{sq}_j$			$\tilde{sn}_j$			$\tilde{w}_j$		
	l	m	u	l	m	u	l	m	u	l	m	u
C_1	0.75	0.85	0.95	0.19	0.32	0.75	0.03	0.06	0.13	0.034	0.092	0.274
C_2	0.55	0.65	0.75	0.37	0.55	1.12	0.06	0.10	0.20	0.065	0.156	0.411
C_3	0.45	0.50	0.50	0.56	0.71	1.23	0.10	0.12	0.22	0.097	0.203	0.452
C_4	0.45	0.50	0.50	0.56	0.71	1.11	0.10	0.12	0.19	0.097	0.203	0.407
C_5	0.25	0.35	0.45	0.56	0.71	1.00	0.10	0.12	0.17	0.097	0.203	0.366
C_6	0.50	0.50	0.50	0.50	0.50	0.50	0.09	0.09	0.09	0.087	0.142	0.183
						5.72	0.478	0.615	1.000			

Table 10
Fuzzy and crisp values of criteria weights obtained using MFI PIPRECIA.

Criterion	$\tilde{w}_j$			w_j^c	w_j
	l	m	u		
C_1	0.034	0.092	0.274	0.133	0.112
C_2	0.065	0.156	0.411	0.211	0.177
C_3	0.097	0.203	0.452	0.251	0.211
C_4	0.097	0.203	0.407	0.236	0.198
C_5	0.097	0.203	0.366	0.222	0.187
C_6	0.087	0.142	0.183	0.138	0.116
				1.190	1.000

narios, especially in cases when the fuzzy weights of the criteria are used in combination with the fuzzy expansion of various MCDM methods.

The attitudes of the respondent obtained by applying the inverse procedure of the MF PIPRECIA method (MFI PIPRECIA) are shown in Table 8, while the details of the calculations are presented in Table 9.

As in the previous case, i.e. in the case of direct relative criteria importance comparisons, transformation of fuzzy criteria weights into crisp values was performed using equations (27) and (28), as is presented in Table 10.

As a result of using the fuzzy MF PIPRECIA-E method, two vectors of fuzzy criteria weights are obtained, which are realistically different to some extent, especially if it is considered that the inverse criteria importance comparisons procedure can be significantly more difficult for the examinee. In the case of transforming fuzzy weights into crisp values,

Table 11
Resulting criteria weights obtained using Eq. (2).

Criterion	w'_j	w''_j	w_j
C_1	0.025	0.112	0.069
C_2	0.177	0.177	0.177
C_3	0.193	0.211	0.202
C_4	0.210	0.198	0.204
C_5	0.210	0.187	0.198
C_6	0.186	0.116	0.151
			1.000

Table 12
Resulting fuzzy criteria weights.

Criterion	$\tilde{w}'_j$			$\tilde{w}''_j$			w_j		
	l	m	u	l	m	u	l	m	u
C_1	0.017	0.059	0.100	0.034	0.092	0.274	0.026	0.076	0.187
C_2	0.034	0.197	0.997	0.065	0.156	0.411	0.050	0.177	0.704
C_3	0.034	0.197	1.108	0.097	0.203	0.452	0.066	0.200	0.780
C_4	0.034	0.197	1.231	0.097	0.203	0.407	0.066	0.200	0.819
C_5	0.031	0.197	1.231	0.097	0.203	0.366	0.064	0.200	0.798
C_6	0.021	0.152	1.119	0.087	0.142	0.183	0.054	0.147	0.651

consistency and criteria weights can be determined as in the PIPRECIA-E method, that is using Eq. (1), for determining consistency of the comparisons, and using Eq. (2) or Eq. (3) for determining resulting criteria weights.

Crisp criteria weights calculated using Eq. (2) are presented in Table 11, with achieved consistency of 0.966.

However, weights obtained by using the MF PIPRECIA-E method do not always have to be transformed into crisp values. In these cases, transformation of the fuzzy weights obtained by the direct and indirect procedure of comparing the relative importance of the criteria into resulting fuzzy criteria weights can be determined by applying Eq. (35), while consistency can be determined using Eq. (34).

Fuzzy criteria weights, calculated by using Eq. (35) are shown in Table 12, where the consistency of the evaluation was 0.994.

4.2. *The Second Numerical Illustration*

The second numerical illustration demonstrates the application of the MF PIPRECIA-E method in a group decision-making environment. In this case, the determination of criteria weight for determining customer satisfaction in traditional Serbian restaurants is also considered, based on the attitudes of three respondents.

The attitudes of the first of the three respondents were previously shown in Tables 3 and 5, while the attitudes of the remaining two respondents are shown in Tables 13, 14, 15 and 16.

It should be emphasized here that the second respondent transformed the linguistic variables into TFNs using the values from the TFN II column.

Table 13
Attitudes of the second respondent obtained using MF PIPRECIA.

Criterion	Linguistic variable	$\tilde{s}_j$		
		l	m	u
C_1				
C_2	MuGI	0.825	0.85	0.875
C_3	SI	0.5	0.5	0.5
C_4	SGI	0.475	0.5	0.525
C_5	SLI	0.475	0.5	0.525
C_6	MoLI	0.325	0.35	0.375

Table 14
Attitudes of the third respondent obtained using MF PIPRECIA.

Criterion	Linguistic variable	$\tilde{s}_j$		
		l	m	u
C_1				
C_2	GI	0.65	0.75	0.85
C_3	SI	0.5	0.5	0.5
C_4	SLI	0.45	0.5	0.5
C_5	SGI	0.5	0.5	0.55
C_6	MoLI	0.25	0.35	0.45

Table 15
Attitudes of the second respondent obtained applying MFI PIPRECIA.

Criterion	Linguistic variable	$\tilde{s}_j$		
		l	m	u
C_1	MuLI	0.125	0.15	0.175
C_2	MoLI	0.325	0.35	0.375
C_3	SLI	0.475	0.5	0.525
C_4	SGI	0.475	0.5	0.525
C_5	MoGI	0.625	0.65	0.675
C_6				

Table 16
Attitudes of the third respondent obtained applying MFI PIPRECIA.

Criterion	Linguistic variable	$\tilde{s}_j$		
		l	m	u
C_1	MoLI	0.25	0.35	0.45
C_2	MoLI	0.25	0.35	0.45
C_3	SLI	0.45	0.5	0.5
C_4	SGI	0.5	0.5	0.55
C_5	MoGi	0.55	0.65	0.75
C_6				

In this case, the second respondent also performed the transformation of linguistic variables into TFNs using the values from the TFN II column.

Table 17
Group decision matrix formed using MF PIPRECIA.

Criterion	$\tilde{s}j$		
	l	m	u
C_1			
C_2	0.741	0.816	0.891
C_3	0.500	0.500	0.516
C_4	0.475	0.500	0.524
C_5	0.475	0.500	0.524
C_6	0.275	0.350	0.425

Table 18
Group decision matrix formed using MFI PIPRECIA.

Criterion	$\tilde{s}j$		
	l	m	u
C_1	0.142	0.216	0.291
C_2	0.275	0.350	0.425
C_3	0.475	0.500	0.524
C_4	0.491	0.500	0.541
C_5	0.574	0.649	0.724
C_6			

Table 19
Calculation details and fuzzy criteria weights obtained using group MF PIPRECIA method.

Criterion	$\tilde{k}_j$			$\tilde{q}_j$			$\tilde{q}'_j$			$\tilde{w}_j$		
	l	m	u	l	m	u	l	m	u	l	m	u
C_1	0.500	0.500	0.500	0.500	0.500	0.500	0.040	0.040	0.040	0.040	0.072	0.104
C_2	0.109	0.184	0.259	0.965	1.358	2.289	0.077	0.108	0.183	0.077	0.195	0.475
C_3	0.484	0.501	0.501	0.964	1.356	2.365	0.077	0.108	0.189	0.077	0.195	0.491
C_4	0.476	0.501	0.525	0.917	1.355	2.487	0.073	0.108	0.199	0.073	0.195	0.516
C_5	0.476	0.501	0.525	0.873	1.354	2.615	0.070	0.108	0.209	0.070	0.194	0.542

The group decision matrices, shown in Tables 17 and 18, were formed by applying the FWA operator, that is, by applying Eq. (10), where the same importance is assigned to all respondents, i.e. $w_j = 0.333$.

The calculation details and fuzzy criteria weights obtained using MF PIPRECIA and MFI PIPRECIA methods are summarized in Tables 19 and 20.

The fuzzy criteria weights obtained by applying the MF PIPRECIA-E method are summarized again in Table 21. Based on the specified fuzzy criteria weights, the resulting fuzzy or crisp criteria weights can be calculated, as well as the consistency of the group pairwise comparisons.

The consistency of group pairwise comparisons, calculated using Eq. (35) and data from Table 21, is very high and it is 0.9732.

Resulting fuzzy criteria weights can be calculated applying Eq. (36), as shown in Table 22.

Table 20
Calculation details and fuzzy criteria weights obtained using group MFI PIPRECIA method.

Criterion	$\tilde{k}_j$			$\tilde{q}_j$			$\tilde{q}'_j$			$\tilde{w}_j$		
	l	m	u	l	m	u	l	m	u	l	m	u
C_1	0.709	0.784	0.858	0.221	0.349	0.637	0.044	0.070	0.128	0.044	0.099	0.226
C_2	0.575	0.650	0.725	0.379	0.547	0.903	0.076	0.110	0.182	0.076	0.155	0.321
C_3	0.476	0.501	0.525	0.549	0.712	1.039	0.110	0.143	0.209	0.110	0.201	0.369
C_4	0.459	0.501	0.509	0.577	0.712	0.988	0.116	0.143	0.199	0.116	0.202	0.351
C_5	0.276	0.351	0.426	0.587	0.713	0.907	0.118	0.143	0.182	0.118	0.202	0.322

Table 21
Fuzzy criteria weights obtained using group MF PIPRECIA-E method.

Criterion	$\tilde{w}'_j$			$\tilde{w}''_j$		
	l	m	u	l	m	u
C_1	0.040	0.072	0.104	0.044	0.099	0.226
C_2	0.077	0.195	0.475	0.076	0.155	0.321
C_3	0.077	0.195	0.491	0.110	0.201	0.369
C_4	0.073	0.195	0.516	0.116	0.202	0.351
C_5	0.070	0.194	0.542	0.118	0.202	0.322
C_6	0.048	0.149	0.471	0.101	0.142	0.178

Table 22
Resulting fuzzy criteria weights obtained using group MF PIPRECIA-E method.

Criterion	$\tilde{w}'_j$			$\tilde{w}''_j$			w_j		
	l	m	u	l	m	u	l	m	u
C_1	0.040	0.072	0.104	0.044	0.099	0.226	0.042	0.085	0.165
C_2	0.077	0.195	0.475	0.076	0.155	0.321	0.077	0.175	0.398
C_3	0.077	0.195	0.491	0.110	0.201	0.369	0.094	0.198	0.430
C_4	0.073	0.195	0.516	0.116	0.202	0.351	0.095	0.198	0.434
C_5	0.070	0.194	0.542	0.118	0.202	0.322	0.094	0.198	0.432
C_6	0.048	0.149	0.471	0.101	0.142	0.178	0.074	0.145	0.325

Crisp criteria weights can be calculated using one of the following Eqs. (26), (28) or (29), and certainly using Eq. (26), separately for $\tilde{w}'_j$ and $\tilde{w}''_j$. After that, using Eq. (2), or Eq. (3), resulting crisp criteria weights can be calculated, as shown in Table 23.

The values of criteria weights w'_j and w''_j are calculated using Eq. (26), while the values in column $(w'_j + w''_j)/2$ were calculated using Eq. (2). The resulting crisp criteria weights calculated using Eq. (27) are shown in column w_j.

A similar calculation of criteria weights applying Eq. (28) and different values of the coefficient α is summarized in Table 24.

From the previous table, it can be noticed that the changes in values of the coefficient α affect the criteria weights. For example, when the value of the coefficient α increases, the weight of criterion C_1 increases, while the weight of criterion C_2 decreases.

Similar calculations of criterion weights can be carried out by varying the value of coefficients α and β in Eq. (29), i.e. coefficient α in Eq. (3), which enables decision makers to examine different decision-making scenarios.

Table 23
Resulting crisp criteria weights obtained using group MF PIPRECIA-E method.

Criterion	$\tilde{w}'_j$			$\tilde{w}''_j$			w'_j	w''_j	$\frac{w'_j+w''_j}{2}$	w_j
	l	m	u	l	m	u				
C_1	0.040	0.072	0.104	0.044	0.099	0.226	0.054	0.104	0.079	0.081
C_2	0.077	0.195	0.475	0.076	0.155	0.321	0.187	0.155	0.171	0.177
C_3	0.077	0.195	0.491	0.110	0.201	0.369	0.191	0.191	0.191	0.198
C_4	0.073	0.195	0.516	0.116	0.202	0.351	0.197	0.187	0.192	0.199
C_5	0.070	0.194	0.542	0.118	0.202	0.322	0.202	0.180	0.191	0.198
C_6	0.048	0.149	0.471	0.101	0.142	0.178	0.168	0.118	0.143	0.148
									$\sum$	1.000

Table 24
Resulting criteria weights obtained using Eq. (28) and different values of coefficient α.

Criterion	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 1$
	w_j				
C_1	0.079	0.080	0.081	0.083	0.086
C_2	0.180	0.179	0.177	0.174	0.170
C_3	0.197	0.197	0.197	0.198	0.198
C_4	0.198	0.198	0.198	0.198	0.198
C_5	0.198	0.198	0.198	0.198	0.198
C_6	0.148	0.148	0.148	0.148	0.149

Table 25
Resulting crisp criteria weights based on calculations performed for each respondent.

Criterion	w'_j			w''_j			w'_j	w''_j	$\frac{w'_j+w''_j}{2}$	w_j
	r_1	r_2	r_3	r_1	r_2	r_3				
C_1	0.025	0.056	0.074	0.112	0.081	0.140	0.052	0.111	0.081	0.083
C_2	0.177	0.195	0.187	0.177	0.136	0.165	0.186	0.159	0.173	0.177
C_3	0.193	0.195	0.187	0.211	0.176	0.197	0.191	0.194	0.193	0.197
C_4	0.210	0.198	0.185	0.198	0.174	0.199	0.198	0.190	0.194	0.199
C_5	0.210	0.201	0.198	0.187	0.173	0.187	0.203	0.182	0.193	0.197
C_6	0.186	0.156	0.169	0.116	0.120	0.116	0.170	0.117	0.144	0.147
									$\sum$	1.000

4.2.1. *Comparison of the Results Obtained by Applying the Group and Individual Approaches for Determining Criteria Weights Using the PIPRECIA-E Method*

As previously stated, there are two basic approaches for determining the resulting crisp criteria weights in cases of group decision making. The first approach is based on complete calculations based on the attitudes of each respondent, and determining average value of resulting criteria weights, as shown in Table 25.

The weights of criteria obtained based on the attitudes of three respondents using the PIPRECIA-E method, that is using direct and indirect comparison procedures, are presented in the previously shown table. In this table, the resulting crisp weights w' and w''

Table 26
Comparison of the resulting crisp criteria weights obtained based on individual and group approach.

Criterion	I	II
C_1	0.083	0.086
C_2	0.177	0.170
C_3	0.197	0.198
C_4	0.199	0.198
C_5	0.197	0.198
C_6	0.147	0.149

are obtained by applying Eq. (26), while the resulting crisp criteria weights are determined using Eq. (2) and Eq. (27).

The resulting crisp criteria weights determined on the basis of calculations carried out on the basis of the attitudes of each respondent and determining mean values of the weights, presented in Table 25, are shown again column I of Table 26, while the resulting crisp criteria weights obtained using the group MF PIPRECIA-E method, presented in Table 24, are again shown in column II of Table 26.

As can be observed from Table 26, the criteria weights obtained by applying the above-mentioned approaches are to a significant extent, which is also confirmed by their similarity, calculated by applying Eq. (1), which is 0.9994. Therefore, the results that are presented in Table 26 show a very high degree of agreement between the weights of the criteria obtained from the individual and group approaches. Observed differences between corresponding weights are relatively small, while similarity coefficient confirms that both approaches produce highly consistent results.

Conclusion

This article presents a modified variant of the PIPRECIA-E method, developed with the aim of keeping values of variables used in calculation procedures in the interval (0, 1], or approximately 1, as well as a fuzzy extension of the modified variant of the PIPRECIA-E method. The proposed modification, as well as its fuzzy extension, was developed primarily with the aim of developing similar fuzzy extensions of the PIPRECIA-E method, such as Intuitionistic, Pythagorean, or Neutrosophic, which can also be highlighted as future directions of development of the proposed approach.

The considered examples confirm that the modified variant of the PIPRECIA-E method gives the same results as the original PIPRECIA-E method, while the considered examples of application of the modified fuzzy PIPRECIA-E method indicate the usability of the proposed fuzzy extension for solving real problems, i.e. determining criteria weights in cases of a group environment, as well as in cases where the decision problem is solved based on the attitudes of one respondent.

In addition, the proposed approach can be considered very flexible, considering that the determination of criteria weights can be done only on the basis of the first approach

of the PIPRECIA-E method, that is, on the basis of the procedure of direct comparison of criteria importance, whether case of using crisp or fuzzy numbers, or whether in cases of decision-making based on attitudes of one or a group of respondents.

For more efficiently using the proposed fuzzy extension of the modified PIPRECIA-E method, an appropriate linguistic scale is also proposed. This linguistic scale enables transformation of each linguistic variable into one of the two predefined triangular fuzzy numbers, which also presents the specificity of the proposed approach. It should also be emphasized that the bounds and modes of these triangular fuzzy numbers are recommended, and that they can be modified with the aim of more precisely expressing the respondents' preferences.

The proposed MF PIPRECIA-E approach can serve as a methodological basis for future developments and extensions by using other types of fuzzy and uncertainty representations. These possible extensions could further increase the flexibility of the MF PIPRECIA-E framework adapted for use in different uncertain decision-making environments.

A. Appendix. The PIPRECIA Method

The computational procedure of the PIPRECIA method, for determining criteria weights of n criteria, can be presented through the following steps:

Step 1. Starting from the second criterion, set the value of s_j for each criterion as follows:

$$\begin{cases} s_j \in (1, 1.9] & \text{when } C_j \succ C_{j-1}, \\ s_j = 1 & \text{when } C_j = C_{j-1}, \\ s_j \in [0.1, 1) & \text{when } C_j \prec C_{j-1}, \end{cases} \tag{A.1}$$

where $j \neq 1$.

Step 2. Transform the value of s_j as follows:

$$sk_j = \begin{cases} 1 & \text{when } j = 1, \\ 2 - s_j & \text{when } j \neq 1, \end{cases} \tag{A.2}$$

$$sq_j = \begin{cases} 1 & \text{when } j = 1, \\ \frac{sq_{j-1}}{sk_j} & \text{when } j \neq 1. \end{cases} \tag{A.3}$$

Step 3. Determine the relative weights of the evaluation criteria as follows:

$$w_j = \frac{sq_j}{\sum_{k=1}^{n} sq_k}. \tag{A.4}$$

B. Appendix. Criteria Weights Determination Using the PIPRECIA Method

The calculation procedure performed using the PIPRECIA method is summarized in Table B.1.

Table B.1
Criteria weights determination using the PIPRECIA method.

Criterion	s_j	sk_j	sq_j	w_j
C_1		1.0	1.0	0.077
C_2	1.6	0.4	2.50	0.191
C_3	1.0	1.0	2.50	0.191
C_4	1.05	0.95	2.63	0.201
C_5	0.95	1.05	2.51	0.192
C_6	0.7	1.3	1.93	0.148

The values in column s_j represent the respondents' attitudes, while the values in a column sk_j, sq_j and w_j are calculated using the formulas Eqs. (A.2), (A.3) and (A.4).

C. Appendix. The PIPRECIA-I Method

The computational procedure of inverse PIPRECIA method for determining criteria weights of n criteria can be represented through the following steps:

Step 1. Starting from the second to last criterion, set the value of s_j for each criterion as follows:

$$\begin{cases} s_j \in (1, 1.9], & \text{when } C_j \succ C_{j+1}, \\ s_j = 1, & \text{when } C_j = C_{j+1}, \\ s_j \in [0.1, 1), & \text{when } C_j \prec C_{j+1}, \end{cases} \tag{C.1}$$

where $j \neq n$.

Step 2. Transform the value of s_j as follows:

$$sk_j = \begin{cases} 1, & \text{when } j = n, \\ 2 - s_j, & \text{when } j \neq n, \end{cases} \tag{C.2}$$

$$sq_j = \begin{cases} 1, & \text{when } j = n, \\ \frac{sq_{j+1}}{sk_j}, & \text{when } j \neq n. \end{cases} \tag{C.3}$$

Step 3. Determine the relative weights of the evaluation criteria as follows:

$$w_j = \frac{sq_j}{\sum_{k=1}^{n} sq_k}. \tag{C.4}$$

D. Appendix. Criteria Weights Determination Using the PIPRECIA-I Method

The calculation procedure performed using the PIPRECIA method is summarized in Table D.1.

The values in column s_j represent the respondents' attitudes, while the values in column sk_j, sq_j and w_j are calculated using the formulas Eqs. (C.2), (C.3) and (C.4).

Table D.1
Criteria weights determination using the PIPRECIA-I method.

Criterion	s_j	sk_j	sq_j	w_j
C_1	0.3	1.70	0.60	0.090
C_2	0.7	1.30	1.03	0.153
C_3	0.95	1.05	1.34	0.199
C_4	1.05	0.95	1.40	0.209
C_5	1.25	0.75	1.33	0.199
C_6		1.0	1.0	0.149

E. Appendix. Criteria Weights Determination Using the Modified PIPRECIA Method

The calculation procedure performed using the M PIPRECIA method is summarized in Table E.1.

Table E.1
Calculation details weights obtained using the modified PIPRECIA method.

Criterion	s_j	sk_j	sq_j	sn_j	w_j
C_1		0.5	0.5	0.08	0.077
C_2	0.8	0.2	1.25	0.19	0.191
C_3	0.5	0.5	1.25	0.19	0.191
C_4	0.525	0.475	1.32	0.20	0.201
C_5	0.475	0.525	1.25	0.19	0.192
C_6	0.35	0.65	0.96	0.15	0.148

From Table E.1. the values used for calculating the criteria weights belong to the interval [0, 1]. Despite the modifications in the calculation procedure of the PIPRECIA method, the weights of the criteria obtained by the modified PIPRECIA method remain the same as in the original version of the PIPRECIA method, which can be seen from Table E.2.

Table E.2
Criteria weights obtained using PIPRECIA and the modified PIPRECIA method.

Criterion	PIPRECIA	Modified PIPRECIA
	w_j	w_j
C_1	0.077	0.077
C_2	0.191	0.191
C_3	0.191	0.191
C_4	0.201	0.201
C_5	0.192	0.192
C_6	0.148	0.148

F. Appendix. Criteria Weights Determination Using the Modified PIPRECIA-I Method

The calculation procedure performed using the MI PIPRECIA method is summarized in Table F.1.

Table F.1
Calculation details weights obtained using the MI PIPRECIA method.

Criterion	s_j	sk_j	sq_j	sn_j	w_j
C_1	0.15	0.85	0.30	0.09	0.090
C_2	0.35	0.65	0.51	0.15	0.153
C_3	0.475	0.53	0.67	0.20	0.199
C_4	0.525	0.48	0.70	0.21	0.209
C_5	0.625	0.38	0.67	0.20	0.199
C_6		0.50	0.5	0.15	0.149

From Table F.1, the values used for calculating the criteria weights belong to the interval [0, 1]. Despite the modifications in the calculation procedure of the inverse PIPRECIA method, the weights of the criteria obtained by the modified inverse PIPRECIA method remain the same as in the original version of the inverse PIPRECIA method, which can be observed from Table F.2.

Table F.2
Criteria obtained weights using inverse PIPRECIA and the modified inverse PIPRECIA method.

Criterion	PIPRECIA	MI PIPRECIA
	w_j	w_j
C_1	0.090	0.090
C_2	0.153	0.153
C_3	0.199	0.199
C_4	0.209	0.209
C_5	0.199	0.199
C_6	0.149	0.149

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