INFORMATICA, 2026, Vol. 0, No. 0, 1–38

© 2026 *Vilnius University*
DOI: https://doi.org/10.15388/26-INFOR648

A Novel Interval-Valued Pythagorean Fuzzy Decision Support Framework for Evaluating Hospital Service Quality

Ali Saeed ALMUFLIH[1,2], Abdullah Mohammed AL FATAIS[1,2], Arunodaya Raj MISHRA[3,4], Erfan Babaee TIRKOLAEE[5,6,*]

[1] *Department of Industrial Engineering, College of Engineering, King Khalid University, P.O. Box 394, Abha 61421, Saudi Arabia*
[2] *Center for Engineering and Technology Innovations, King Khalid University, Abha 61421, Saudi Arabia*
[3] *Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences (SIMATS), Chennai, TN, India*
[4] *Jadara University Research Center, Jadara University, Irbid 21110, Jordan*
[5] *Department of Industrial Engineering, Istinye University, Istanbul, Turkey*
[6] *Department of Industrial Engineering and Management, Yuan Ze University, Taoyuan, Taiwan*
e-mail: asalmuflih@kku.edu.sa, alftas@kku.edu.sa, arunodayaraj.mishra@ostimteknik.edu.tr, erfan.babaee@istinye.edu.tr

Received: February 2026; accepted: October 2026

Abstract. This study proposes an integrated decision support framework for evaluating hospital service quality under uncertainty using Interval-Valued Pythagorean Fuzzy (IVPF) information. It combines an improved entropy measure for expert weighting, WENSLO-RANCOM method for criteria weighting, and an IVPF extension of CRADIS method for hospital ranking. The framework is applied to five hospitals in Patna, India, using four experts and eighteen criteria. Public transport, ambulance services, and medication expertise emerge as the most important factors. Hospital-2 ranks first overall. Sensitivity and comparative analyses confirm the robustness of the results, while the framework also supports resource allocation and service improvement planning.

Key words: hospital service quality, decision support framework, interval-valued Pythagorean fuzzy sets, entropy, score function.

1. Introduction

The healthcare system is an indispensable pillar of any country's economic growth. Diverse healthcare needs, increasing public expectations, and more ambitious health objectives are placing greater demands on health systems to achieve improved health outcomes (Rehman *et al.*, 2023). Delivering health services that align with patients' needs and expectations enhances organizations' ability to remain competitive in today's healthcare sector.

*Corresponding author.

Consequently, enhancing healthcare quality has become a central objective for healthcare organizations worldwide, as it supports service improvement, financial sustainability, and patient satisfaction regarding the standard of care (Darzi *et al.*, 2023). Moreover, enhanced patient care and support services within hospitals, beyond the treatment of medical conditions, significantly enhance patients' trust in hospital management while keeping costs under control. In recent years, this sector has undergone transformative changes, driven by technological advancements, data ecosystems, evolving patient expectations, and increased regulatory pressures (Khan *et al.*, 2023). Ensuring patient satisfaction while delivering accurate diagnoses represents a key challenge that hospitals must address in their efforts to improve healthcare service quality.

Since healthcare organizations are directly responsible for saving lives, the ongoing demand for improved outcomes and high-quality healthcare services is critically important for developing effective organizational policies for reducing premature mortality and preventable patient harm. Healthcare sectors, as one of the leading service sectors, recognize that the service quality of hospitals is particularly relevant to the promotion of the healthcare market in today's competitive environment (Ferreira *et al.*, 2023; Zan *et al.*, 2026). The survivability of healthcare institutions depends heavily on clients utilizing the services and determining that they adequately satisfy their immediate needs (Sanıl and Eminer, 2021). In developing countries, customers of the healthcare sector are increasingly aware of their right to receive quality healthcare (Darzi *et al.*, 2023). Therefore, delivering quality services within healthcare organizations has become an increasingly important priority in the nation's booming health industry. According to Cui and Tan (2025), healthcare providers need to deliver high-quality services that offer improved customer retention, higher customer satisfaction, enlarged market shares, and increased profit margins. Additionally, enhancing service quality while reducing costs provides healthcare institutions with a competitive advantage in distinguishing themselves from their competitors (Khan *et al.*, 2023; Amit *et al.*, 2026).

Patient satisfaction is a key indicator in assessing hospital quality, as it reflects how effectively healthcare staff meet patients' needs and expectations. This satisfaction is largely shaped by patients' perceptions of the quality of care and services they receive (Farrokhi *et al.*, 2023). High levels of hospital service quality enhance patient satisfaction, thereby increasing patients' willingness to establish and maintain long-term relationships with healthcare providers (Zehra *et al.*, 2025). Patient-perceived healthcare quality, which covers dimensions, namely patient relationship management, patient management system, and patient-physician relationship, has been recognized as a significant factor in measuring patient satisfaction (Rehman *et al.*, 2023; Mini *et al.*, 2025). Assessment of patient-perceived hospital service quality is taken into account as a complex Multi-Criteria Decision-Making (MCDM) problem, as it involves diverse criteria and uncertain information.

In real-life decision-making situations, the DMs may encounter problems in articulating their preferences as a numerical value (Alkan and Kahraman, 2025). To solve these problems, Zadeh (1965) proposed the theoretical notion of Fuzzy Set (FS) to capture uncertainty arising from human subjectivity. It represents the degree to which an

object belongs to a set through membership functions defined on a continuous scale from 0 to 1. To address the limitations of FS theory, Atanassov (1986) presented the Intuitionistic Fuzzy Set (IFS), which is denoted by the Membership Degree (MD) 'μ' and Non-Membership Degree (NMD) 'ν', with constraint $0 \leqslant \mu + \nu \leqslant 1$. Further, Yager (2014) put forward the notion of Pythagorean Fuzzy Set (PFS), which extends the theory of IFS by replacing the linear constraint $0 \leqslant \mu+\nu \leqslant 1$ subject to $0 \leqslant \mu^2+\nu^2 \leqslant 1$. Accordingly, PFS offers decision experts a broader range for expressing fuzzy information.

Afterward, Peng and Yang (2016) defined the Interval-Valued Pythagorean Fuzzy Set (IVPFS) with its own mathematical procedures. IVPFS extends the concept of PFS by expressing membership degrees as intervals instead of precise values; therefore, it can tackle a wider range of uncertain information. Considering the effectiveness of IVPFSs, there has been an increasing interest among scholars in utilizing them for solving real-life problems. For instance, Kose *et al.* (2022) proposed a hybrid MCDM framework for ergonomic assessment of the setup process under a single-minute exchange of dies. In this regard, they integrated the Analytic Hierarchy Process (AHP) method and TOPSIS within the context of IVPFSs and applied it to a manufacturing problem. Alhamzi *et al.* (2023) presented a score function for ordering Interval-Valued Pythagorean Fuzzy Numbers (IVPFNs). Additionally, they proposed a set of Dombi operators for aggregating IVPFNs into a single IVPFN. Within the context of IVPFSs, Rani *et al.* (2023) developed a hybrid Operational Competitiveness Rating Analysis (OCRA) approach based on a new score function, interaction operators, and cross-entropy, and employed to a case study focused on metaverse integration options in the transportation sharing economy. On the basis of the proposed membership score, Alrasheedi and Jeevaraj (2023) defined a total ordering principle on IVPFNs to deal with MCDM problems. Mishra *et al.* (2023) described the degree of fuzziness of IVPFS using a new entropy measure. Further, they aggregated a set of IVPFNs into an IVPFN with the use of fairly aggregation operator. Moreover, they introduced a hybrid Multi-Attribute Ideal-Real Comparative Analysis (IVPF-MAIRCA) method for assessing the blockchain-based healthcare supply chain.

Recently, Arisoy *et al.* (2025) proposed four score functions for distinguishing different IVPFNs with some interesting mathematical axioms and presented a hybrid Multi-Criteria Group Decision-Making (MCGDM) model for assessing the best agricultural logistics system in developing countries. Extending the Decision-Making Trial and Evaluation Laboratory (DEMATEL) approach in the IVPFS environment, Alam *et al.* (2025) offered an integrated MCDM framework to mitigate traffic congestion, which helps policymakers in long-term urban planning and environmental sustainability. Based on triangular divergence, Liu *et al.* (2025) suggested a distance measure on IVPFSs and developed a collective IVPF distance-based extended inferior ratio approach for solving the green supplier selection problem. With the use of an integrated IVPF-MCDM problem, Oh *et al.* (2025) evaluated five hydrogen storage technologies and identified underground storage as the optimal solution for large-scale hydrogen storage under Saudi Arabia's specific conditions.

As one of the most recent MCDM techniques, the Compromise Ranking of Alternatives from Distance to Ideal Solution (CRADIS) (Puška *et al.*, 2022) prioritizes alternatives considering their closeness to an ideal solution. The approach combines normalized

and weighted distance calculations to derive a compromise ranking for each option. Recently, many studies on extended CRADIS methods with different extended versions of FS theory have been conducted to rank the alternatives under uncertain environments. For instance, Yuan *et al.* (2023) offered a hybrid picture fuzzy Jensen-Shannon divergence-based CRADIS approach and applied it to rank six alternatives across four evaluation criteria, including resource consumption, carbon emission, green production, and green technology. Demir and Moslem (2024) integrated the fuzzy CRADIS method with a fuzzy preference selection index for evaluating the most appropriate medical waste treatment techniques, considering technological, economic, environmental, and social factors. By combining the CRADIS technique with the entropy method, Anjum *et al.* (2025) introduced a combined intuitionistic fuzzy MCDM framework to optimize a range of energy-efficient solutions in smart manufacturing. Işık and Adalar (2025) suggested a fuzzy CRADIS model for assessing and ranking the environmental, social, and governance performance of non-life insurance firms in Turkey. Thus far, the literature does not report any extensions of the CRADIS method under the setting of IVPFSs.

Considering the effectiveness of the CRADIS method and IVPFSs, this research offers a hybrid decision-making methodology and employs it to evaluate hospital service quality. In this framework, we first derive the DMs' weights through the entropy method and incorporate them into the IVPF-weighted averaging operator; we aggregate the individual IVPF opinions of each expert to appraise the quality of service of hospitals. Next, we derive the weights of criteria with the help of an integrated objective-subjective model, consisting of Weights by ENvelope and SLOpe (WENSLO) and RANking COMparison (RANCOM), respectively. Based on the combination of the above steps, the algorithm of CRADIS is to determine alternatives' ranking in the context of IVPF information. Consequently, a novel entropy measure, a modified score function, and an improved distance measure are developed in the context of IVPFSs, followed by comparative analyses. The suggested entropy technique is used to derive the DMs' weights, while the score function is employed to normalize the input information during the computation of criteria weights through the WENSLO technique. On the flip side, the suggested distance formula is employed to find the closeness of each option to the IVPF-ideal and IVPF anti-ideal solutions via the CRADIS technique.

The originality of this work is primarily reflected in the following:

i. A novel IVPF-entropy measure is offered in this work for measuring the degree of fuzziness of an IVPFS. Further, the DMs' weights are obtained based on the suggested entropy measure.
ii. To compare different IVPFNs, this research proposes an enhanced IVPF-score function, which relaxes the weaknesses of existing IVPF-score functions. Moreover, the proposed score formula is utilized to find the normalized decision matrix in the WENSLO model for criteria weighting.
iii. To distinguish the IVPFSs, a modified distance measure is proposed for IVPFSs, followed by comparison with existing IVPF-score functions. It is further utilized to find the dissimilarity degrees between alternatives and ideal solutions.

iv. To treat the MCGDM problems, a hybrid IVPF-CRADIS method is developed by merging its algorithm with the entropy method for DMs' weights and an integrated WENSLO-RANCOM model for criteria weighting within the setting of IVPFSs.
v. To demonstrate the efficiency of the developed methodology, it is implemented in a case study of a hospital service quality evaluation problem under an IVPF environment.

The rest of the manuscript is structured as follows. Section 2 reviews the literature of hospital service quality assessment. Section 3 first recalls the concepts related to IVPFSs and further introduces the novel entropy measure, a modified score function, and an improved distance measure in the context of IVPFSs, followed by comparative discussions. Section 4 elaborates the application steps of an incorporated IVPF-WENSLO-RANCOM-CRADIS methodology for treating MCGDM problems. Section 5 implements the developed methodology to a hospital service quality selection problem. Section 6 presents the sensitivity and comparative analyses to reveal the stability and robustness of the decision support framework, respectively. Section 7 concludes the research and recommends several directions for future work.

2. Literature Survey

This section explores the existing works on service quality of health organizations using different decision-making methods. Akdag *et al.* (2014) presented an MCDM framework to evaluate service quality of Turkish hospitals. In this regard, they combined AHP, TOPSIS, and Yager's min-max approach within the setting of fuzzy information and later implemented it in an empirical study of the healthcare service quality evaluation problem of four B-class hospitals of Istanbul. Employing a fuzzy AHP-TOPSIS model, Shafii *et al.* (2016) evaluated the service quality of teaching hospitals in Yazd, Iran, considering six dimensions of assurance, responsiveness, tangibles, security, patient orientation, and health communication. Liu *et al.* (2019) presented a Grey Relational Analysis (GRA) for appraising the healthcare service quality of hospitals. Based on the obtained outcomes, they determined that the GRA decreases the latency in the assessment process and increases the efficacy range of tools used for healthcare services. Utilizing the Pythagorean fuzzy AHP model, Yucesan and Gul (2020) specified the weights of service quality assessment factors of 32 hospitals. Based on this, they determined the priorities of healthcare institutions through the Pythagorean fuzzy TOPSIS method and offered the opportunity to make evaluations in view of DMs' subjective opinions.

Babroudi *et al.* (2021) measured the customer perceptions of hospital service quality. For this purpose, they suggested an incorporated framework model that integrates Z-number with fuzzy cognitive map to assess healthcare service quality. Sivakumar *et al.* (2022) integrated a multi-objective linear programming model with BWM and Multi-Actor Multi-Criteria Assessment (MAMCA) for enhancing the patient service quality in the intensive care unit. Their findings provided an effective solution that can be utilized to advance the quality of services for intensive care unit patients during the COVID-19

pandemic. By combining the fuzzy rule-based system and TOPSIS model, Karasan *et al.* (2022) suggested a Pythagorean fuzzy-based MCDM tool to assess the service quality performance of ten clinics in a private hospital. With a focus on patient-centred services, Khan *et al.* (2023) identified the quality of services of four hospitals during the recent pandemic. For this purpose, they used a fuzzy BWM and implemented it in a case study of a service quality factors assessment problem, taking into account the patients' perceptions and expectations of the services.

Considering a variety of criteria, Li *et al.* (2023) presented a fuzzy rough copula Bayesian network to assess the service quality of hospitals under an uncertain environment, which deals with the subjective uncertainty of DMs' opinions. Using fuzzy logic and decision modelling, Rehman *et al.* (2023) assessed hospital service quality considering numerous criteria and dimensions based on patients' viewpoints. Their findings suggested that the dynamic nature of hospitals and varying patient expectations require periodic service quality evaluations. To evaluate the quality of healthcare services, Alshouha *et al.* (2024) presented a fuzzy recommendation procedure based on the Quality Function Deployment (QFD) approach. By means of patient opinions and preferences, their findings indicated a step toward a more adapted and operative healthcare structure selection using fuzzy information.

Accounting for smart medical systems, Wang and Shao (2024) integrated the BWM and VIKOR models for assessing the level of medical services offered by hospitals. Cui and Tan (2025) suggested a two-phase fuzzy MCDM model for periodic service quality enhancements in hospitals, in which fuzzy logic was utilized to enhance the accuracy of decisions in hospital quality management. Their method aimed to provide a novel and inspiring mode to advance the effectiveness and ease of healthcare services for patients. Through a comprehensive literature review, Datt *et al.* (2025) identified the principal quality dimensions within the healthcare services domain. Furthermore, they developed an evaluation model for healthcare centres that highlights their capacity to enhance service quality and increase patient satisfaction.

In accordance with the existing literature, we identify that the previous decision-making models ignore the significance of decision experts' opinions during the assessment of hospital service quality. Additionally, these studies overlook the importance of multiple experts in making decisions on the quality of healthcare services. An additional research gap concerns the insufficient consideration of service quality indicators from both objective and subjective perspectives. To handle these gaps, this work suggests a fuzzy integrated decision support framework by incorporating the entropy method for DMs weighting and an integrated WENSLO-RANCOM model for criteria weighting with the steps of the CRADIS approach under the setting of IVPFSs.

3. Proposed Entropy, Score, and Distance Formulae in the Context of IVPFSs

This section begins with a brief overview of the basic notions of IVPFSs, followed by the development of an improved score function, entropy measure, and distance measure.

3.1. *Preliminaries*

Definition 1. Peng and Yang (2016) presented a mathematical definition of an IVPFS P on a fixed universal set $L = \{l_1, l_2, \ldots, l_r\}$, and given as

$$B = \left\{\left(l_i, \left(\left[\mu_B^-(l_i), \mu_B^+(l_i)\right], \left[\nu_B^-(l_i), \nu_B^+(l_i)\right]\right)\right) : l_i \in L\right\}, \tag{1}$$

where $0 \leqslant \mu_B^-(l_i) \leqslant \mu_B^+(l_i) \leqslant 1$, $0 \leqslant \nu_B^-(l_i) \leqslant \nu_B^+(l_i) \leqslant 1$ and $0 \leqslant (\mu_B^+(l_i))^2 + (\nu_B^+(l_i))^2 \leqslant 1$. Here, $\mu_B(l_i) = [\mu_B^-(l_i), \mu_B^+(l_i)]$ and $\nu_B(l_i) = [\nu_B^-(l_i), \nu_B^+(l_i)]$ stand for the interval-valued MD and NMD of an element l_i in B, respectively. The function $\pi_B(l_i) = [\pi_B^-(l_i), \pi_B^+(l_i)]$ denotes the degree of hesitancy/indeterminacy l_i to B, where $\pi_B^-(l_i) = \sqrt{1 - (\mu_B^+(l_i))^2 - (\nu_B^+(l_i))^2}$ and $\pi_B^+(l_i) = \sqrt{1 - (\mu_B^-(l_i))^2 - (\nu_B^-(l_i))^2}$. Peng and Yang (2016) defined the term $([\mu_B^-(l_i), \mu_B^+(l_i)], [\nu_B^-(l_i), \nu_B^+(l_i)])$ as an interval-valued Pythagorean fuzzy value (IVPFV)/interval-valued Pythagorean fuzzy number (IVPFN) and displayed in this work as $\delta = ([\mu^-, \mu^+], [\nu^-, \nu^+])$, which satisfies $0 \leqslant (\mu^+) + (\nu^+) \leqslant 1$.

Definition 2. Let $\delta_1 = ([\mu_1^-, \mu_1^+], [\nu_1^-, \nu_1^+])$ and $\delta_2 = ([\mu_2^-, \mu_2^+], [\nu_2^-, \nu_2^+])$ be two IVPFNs. The main operations of IVPFNs are defined as in Peng and Yang (2016):

(i) $\delta_1 \oplus \delta_2$
$= \left(\left[\sqrt{(\mu_1^-)^2 + (\mu_2^-)^2 - (\mu_1^-)^2(\mu_2^-)^2}, \sqrt{(\mu_1^+)^2 + (\mu_2^+)^2 - (\mu_1^+)^2(\mu_2^+)^2}\right], \left[\nu_1^- \nu_2^-, \nu_1^+ \nu_2^+\right]\right)$,

(ii) $\delta_1 \otimes \delta_2$
$= \left(\left[\mu_1^- \mu_2^-, \mu_1^+ \mu_2^+\right], \left[\sqrt{(\nu_1^-)^2 + (\nu_2^-)^2 - (\nu_1^-)^2(\nu_2^-)^2}, \sqrt{(\nu_1^+)^2 + (\nu_2^+)^2 - (\nu_1^+)^2(\nu_2^+)^2}\right]\right)$,

(iii) $\lambda\delta_i = \left(\left[\sqrt{1 - \left(1 - \left(\mu_i^-\right)^2\right)^\lambda}\sqrt{1 - \left(1 - \left(\mu_i^+\right)^2\right)^\lambda}\right], \left[\left(\nu_i^-\right)^\lambda, \left(\nu_i^+\right)^\lambda\right]\right)$
$(\lambda > 0; i = 1, 2)$,

(iv) $(\delta_i)^\lambda = \left(\left[\left(\mu_i^-\right)^\lambda, \left(\mu_i^+\right)^\lambda\right], \left[\sqrt{1 - \left(1 - \left(\nu_i^-\right)^2\right)^\lambda}, \sqrt{1 - \left(1 - \left(\nu_i^+\right)^2\right)^\lambda}\right]\right)$
$(\lambda > 0; i = 1, 2)$,

(v) $(\delta_i)^c = \left(\left[\nu_i^-, \nu_i^+\right], \left[\mu_i^-, \mu_i^+\right]\right) \quad (i = 1, 2)$.

Definition 3. For an IVPFN $\delta = ([\mu^-, \mu^+], [\nu^-, \nu^+])$, Peng and Yang (2016) defined the score function as

$$S_P(\delta) = \frac{1}{2}\left(\left(\mu^-\right)^2 + \left(\mu^+\right)^2 - \left(\nu^-\right)^2 - \left(\nu^+\right)^2\right), \quad \text{where } S_P(\delta) \in [-1, 1]. \tag{2}$$

With the help of this setting, Peng and Yang (2016) presented a comparison rule for two IVPFNs $\delta_1 = ([\mu_1^-, \mu_1^+], [\nu_1^-, \nu_1^+])$ and $\delta_2 = ([\mu_2^-, \mu_2^+], [\nu_2^-, \nu_2^+])$, given as

- If $S_P(\delta_1) < S_P(\delta_2)$, then δ_1 is considered smaller than δ_2, and denoted as $\delta_1 < \delta_2$,
- If $S_P(\delta_1) > S_P(\delta_2)$, then δ_1 is considered larger than δ_2, and denoted as $\delta_1 > \delta_2$,
- If $S_P(\delta_1) = S_P(\delta_2)$, then θ_1 is considered similar as θ_2, and denoted as $\delta_1 = \delta_2$.

For any two IVPFNs $\delta_1 = ([0.3, 0.4], [0.3, 0.4])$ and $\delta_2 = ([0.4, 0.5], [0.4, 0.5])$, the score function defined by Eq. (2) gives $S_P(\delta_1) = 0$ and $S_P(\delta_2) = 0$. In this context, an expert cannot discriminate the difference between two IVPFNs. To this aim, Peng and Yang (2016) defined the accuracy function, given by

$$A_P(\delta) = \frac{1}{2}\left(\left(\mu^-\right)^2 + \left(\mu^+\right)^2 + \left(\nu^-\right)^2 + \left(\nu^+\right)^2\right), \quad \text{where } A_P(\delta) \in [0, 1]. \tag{3}$$

Definition 4. Let δ_i $(i = 1, 2, \ldots, r)$ be a set of IVPFNs and $\kappa = (\kappa_1, \kappa_2, \ldots, \kappa_r)^T$ be the weight vector of δ_i $(i = 1, 2, \ldots, r)$, where $\kappa_i \in [0, 1]$ and $\sum_{i=1}^{r} \kappa_i = 1$. An Interval-Valued Pythagorean Fuzzy Weighed Averaging (IVPFWA) operator is then defined as in Peng and Yang (2016):

$$\begin{aligned} IVPFWA(\delta_1, \delta_2, \ldots, \delta_r) &= \bigoplus_{i=1}^{r} (\kappa_i \delta_i) \\ &= \left(\left[1 - \prod_{i=1}^{r} \left(1 - \left(\mu_{\delta_i}^-\right)^2\right)^{\kappa_i}, 1 - \prod_{i=1}^{r} \left(1 - \left(\mu_{\delta_i}^+\right)^2\right)^{\kappa_i}\right], \left[\prod_{i=1}^{r} \left(\nu_{\delta_i}^-\right)^{2\kappa_i}, \prod_{i=1}^{r} \left(\nu_{\delta_i}^+\right)^{2\kappa_i}\right]\right). \end{aligned} \tag{4}$$

Additionally, Peng and Yang (2016) defined an Interval-Valued Pythagorean Fuzzy Weighed Geometric (IVPFWG) operator, given as

$$\begin{aligned} IVPFWG(\delta_1, \delta_2, \ldots, \delta_r) &= \bigotimes_{i=1}^{r} \left(\delta_i^{\kappa_i}\right) \\ &= \left(\left[\prod_{i=1}^{r} \left(\mu_{\delta_i}^-\right)^{2\kappa_i}, \prod_{i=1}^{r} \left(\mu_{\delta_i}^+\right)^{2\kappa_i}\right], \left[1 - \prod_{i=1}^{r} \left(1 - \left(\nu_{\delta_i}^-\right)^2\right)^{\kappa_i}, 1 - \prod_{i=1}^{r} \left(1 - \left(\nu_{\delta_i}^+\right)^2\right)^{\kappa_i}\right]\right). \end{aligned} \tag{5}$$

Definition 5. Let $B \in IVPFS(L)$, then a real-valued function $E : IVPFS(L) \to [0, 1]$ is called an IVPF-entropy if it satisfies the following axioms (Peng and Li, 2019):

($\mathbf{a_1}$) $E(B) = 0$ iff B is a crisp set,
($\mathbf{a_2}$) $E(B) = 1$ iff $\mu_B^-(l_i) = \nu_B^-(l_i)$ and $\mu_B^+(l_i) = \nu_B^+(l_i)$, for each $l_i \in L$,
($\mathbf{a_3}$) $E(B) = E(B^c)$,
($\mathbf{a_4}$) $E(B) \leqslant E(N)$ if $B \subseteq N$ with $\mu_N^-(l_i) \leqslant \nu_N^-(l_i)$ and $\mu_N^+(l_i) \leqslant \nu_N^+(l_i)$, for each $l_i \in L$, or $B \supseteq N$ with $\mu_N^-(l_i) \geqslant \nu_N^-(l_i)$ and $\mu_N^+(l_i) \geqslant \nu_N^+(l_i)$, for each $l_i \in L$, where $N \in IVPFS(L)$.

Definition 6. Let $B, N \in IVPFSs(L)$. Then a real-valued mapping $d : IVPFSs(L) \times IVPFSs(L) \to [0, 1]$ is a real-valued function, satisfying the following requirements (Zhang, 2016):

($\mathbf{p_1}$) $0 \leqslant d(B, N) \leqslant 1$,
($\mathbf{p_2}$) $d(B, N) = d(N, B)$,
($\mathbf{p_3}$) $d(B, N) = 0$ if and only if $B = N$,
($\mathbf{p_4}$) If $B \subseteq N \subseteq G$, then $d(B, G) \geqslant d(B, N)$ and $d(B, G) \geqslant d(N, G)$, where $G \in IVPFS(L)$.

3.2. *Proposed Entropy Measure for IVPFS*

The concept of entropy plays a fundamental role in measuring the degree of fuzziness and uncertainty within FS theory and its extensions (Saha *et al.*, 2025). As an important research topic, entropy has received significant scholarly attention over the past several decades. For IVPFS, Peng and Li (2019) firstly proposed an IVPF-entropy and its applications. Besides, some scholars put forward different entropy formulae for IVPFS (e.g., Moazzamigodarzi *et al.*, 2025). Compared with FS, IFS, and PFS, there are not as many studies on the entropy of IVPFS. Unlike existing studies, this paper not only gives the formula of IVPF-entropy, but also presents the use of the suggested measure in determining experts' weights.

Theorem 1. *For $B \in IVPFS(L)$, the function*

$$E(B) = \frac{1}{r}\sum_{i=1}^{r}\left(\frac{2 + (\pi_B^-(l_i))^2 + (\pi_B^+(l_i))^2 - |(\mu_B^-(l_i))^2 - (\nu_B^-(l_i))^2| - |(\mu_B^+(l_i))^2 - (\nu_B^+(l_i))^2|}{2 + (\pi_B^-(l_i))^2 + (\pi_B^+(l_i))^2}\right) \tag{6}$$

is an entropy measure for IVPFS 'B'.

Proof. To prove this theorem, we need to prove the axioms (a_1)–(a_4) of Definition 5.

(a_1) Let B be a crisp set. It implies $\mu_B^-(l_i) = 1 = \mu_B^+(l_i)$, $\nu_B^-(l_i) = 0 = \nu_B^+(l_i)$ or $\mu_B^-(l_i) = 0 = \mu_B^+(l_i)$, $\nu_B^-(l_i) = 1 = \nu_B^+(l_i)$, and therefore, $\pi_B^-(l_i) = 0 = \pi_B^+(l_i)$, $\forall l_i \in L$. In this case, Eq. (6) becomes $E(B) = 0$.

On the other hand, assume that $E(P) = 0$, then from Eq. (6), we have

$$\begin{aligned}
&\frac{2 + \left(\pi_B^-(l_i)\right)^2 + \left(\pi_B^+(l_i)\right)^2 - \left|(\mu_B^-(l_i))^2 - (\nu_B^-(l_i))^2\right| - \left|(\mu_B^+(l_i))^2 - (\nu_B^+(l_i))^2\right|}{2 + \left(\pi_B^-(l_i)\right)^2 + \left(\pi_B^+(l_i)\right)^2} = 0 \quad \forall l_i \in L,\\
\Rightarrow \quad &\left|\left(\mu_B^-(l_i)\right)^2 - \left(\nu_B^-(l_i)\right)^2\right| + \left|\left(\mu_B^+(l_i)\right)^2 - \left(\nu_B^+(l_i)\right)^2\right|\\
&\quad = 2 + \left(\pi_B^-(l_i)\right)^2 + \left(\pi_B^+(l_i)\right)^2, \quad \forall l_i \in L.
\end{aligned} \tag{7}$$

As $B \in IVPFS(L)$, then $0 \leqslant \mu_B^-(l_i) \leqslant \mu_B^+(l_i) \leqslant 1, 0 \leqslant \nu_B^-(l_i) \leqslant \nu_B^+(l_i) \leqslant 1$ and $0 \leqslant (\mu_B^+(l_i))^2 + (\nu_B^+(l_i))^2 \leqslant 1$. Therefore, the only possible solution of Eq. (7) is $\mu_B^-(l_i) = 1 = \mu_B^+(l_i)$, $\nu_B^-(l_i) = 0 = \nu_B^+(l_i)$ or $\mu_B^-(l_i) = 0 = \mu_B^+(l_i)$, $\nu_B^-(l_i) = 1 = \nu_B^+(l_i)$ and $\pi_B^-(l_i) = 0 = \pi_B^+(l_i)$, $\forall l_i \in L$. It implies that 'B' is a crisp set.

(a_2) Let $\mu_B^-(l_i) = \nu_B^-(l_i)$ and $\mu_B^+(l_i) = \nu_B^+(l_i)$, $\forall l_i \in L$. Then, from Eq. (6), we get $E(B) = 1$. On the other hand, assume that $E(B) = 1$, then from Eq. (6), we have

$$\begin{aligned}
&\frac{2 + \left(\pi_B^-(l_i)\right)^2 + \left(\pi_B^+(l_i)\right)^2 - \left|(\mu_B^-(l_i))^2 - (\nu_B^-(l_i))^2\right| - \left|(\mu_B^+(l_i))^2 - (\nu_B^+(l_i))^2\right|}{2 + \left(\pi_B^-(l_i)\right)^2 + \left(\pi_B^+(l_i)\right)^2} = 1, \quad \forall l_i \in L,\\
\Rightarrow \quad &\left|\left(\mu_B^-(l_i)\right)^2 - \left(\nu_B^-(l_i)\right)^2\right| + \left|\left(\mu_B^+(l_i)\right)^2 - \left(\nu_B^+(l_i)\right)^2\right| = 0, \quad \forall l_i \in L.
\end{aligned} \tag{8}$$

Thus, the possible solution of Eq. (8) is $\mu_B^-(l_i) = \nu_B^-(l_i)$ and $\mu_B^+(l_i) = \nu_B^+(l_i)$, for each $l_i \in L$.

(a_3) Let $B \in IVPFS(L)$, where $B = \{(l_i, ([\mu_B^-(l_i), \mu_B^+(l_i)], [\nu_B^-(l_i), \nu_B^+(l_i)])) : l_i \in L\}$. Then the complement of B is denoted as $B^c = \{(l_i, ([\nu_B^-(l_i), \nu_B^+(l_i)], [\mu_B^-(l_i), \mu_B^+(l_i)])) : l_i \in L\}$. Thus, from Eq. (6), we have $E(B) = E(B^c)$.

(a4) For $B, N \in IVPFSs(L)$, let $B \subseteq N$ with $\mu_N^-(l_i) \leqslant \nu_N^-(l_i)$ and $\mu_N^+(l_i) \leqslant \nu_N^+(l_i)$, for each $l_i \in L$. It follows that $\mu_B^-(l_i) \leqslant \mu_N^-(l_i) \leqslant \nu_N^-(l_i) \leqslant \nu_B^-(l_i)$ and $\mu_B^+(l_i) \leqslant \mu_N^+(l_i) \leqslant \nu_N^+(l_i) \leqslant \nu_B^+(l_i)$, for each $l_i \in L$.

Since $|(\mu_B^-(l_i))^2 - (\nu_B^-(l_i))^2| \geqslant |(\mu_N^-(l_i))^2 - (\nu_N^-(l_i))^2|$ and $|(\mu_B^+(l_i))^2 - (\nu_B^+(l_i))^2| \geqslant |(\mu_N^+(l_i))^2 - (\nu_N^+(l_i))^2|$, it implies that $-|(\mu_B^-(l_i))^2 - (\nu_B^-(l_i))^2| \leqslant -|(\mu_N^-(l_i))^2 - (\nu_N^-(l_i))^2|$ and $-|(\mu_B^+(l_i))^2 - (\nu_B^+(l_i))^2| \leqslant -|(\mu_N^+(l_i))^2 - (\nu_N^+(l_i))^2|$, and therefore, $-|(\mu_B^-(l_i))^2 - (\nu_B^-(l_i))^2| - |(\mu_B^+(l_i))^2 - (\nu_B^+(l_i))^2|$ is also less than or equal to $-|(\mu_N^-(l_i))^2 - (\nu_N^-(l_i))^2| - |(\mu_N^+(l_i))^2 - (\nu_N^+(l_i))^2|$,

$$
\begin{aligned}
&\Rightarrow \begin{pmatrix} 2 + \left(1 - \left(\mu_B^+(l_i)\right)^2 - \left(\nu_B^+(l_i)\right)^2\right) + \left(1 - \left(\mu_B^-(l_i)\right)^2 - \left(\nu_B^-(l_i)\right)^2\right) \\ -\left|\left(\mu_B^-(l_i)\right)^2 - \left(\nu_B^-(l_i)\right)^2\right| - \left|\left(\mu_B^+(l_i)\right)^2 - \left(\nu_B^+(l_i)\right)^2\right| \end{pmatrix} \\
&\leqslant \begin{pmatrix} 2 + \left(1 - \left(\mu_N^+(l_i)\right)^2 - \left(\nu_N^+(l_i)\right)^2\right) + \left(1 - \left(\mu_N^-(l_i)\right)^2 - \left(\nu_N^-(l_i)\right)^2\right) \\ -\left|\left(\mu_N^-(l_i)\right)^2 - \left(\nu_N^-(l_i)\right)^2\right| - \left|\left(\mu_N^+(l_i)\right)^2 - \left(\nu_N^+(l_i)\right)^2\right| \end{pmatrix}, \quad \forall l_i \in L, \\
&\Rightarrow \left(2 + \left(\pi_B^-(l_i)\right)^2 + \left(\pi_B^+(l_i)\right)^2 - \left|\left(\mu_B^-(l_i)\right)^2 - \left(\nu_B^-(l_i)\right)^2\right| - \left|\left(\mu_B^+(l_i)\right)^2 - \left(\nu_B^+(l_i)\right)^2\right|\right) \\
&\leqslant \left(2 + \left(\pi_N^-(l_i)\right)^2 + \left(\pi_N^+(l_i)\right)^2 - \left|\left(\mu_N^-(l_i)\right)^2 - \left(\nu_N^-(l_i)\right)^2\right| - \left|\left(\mu_N^+(l_i)\right)^2 - \left(\nu_N^+(l_i)\right)^2\right|\right), \\
&\quad \forall l_i \in L.
\end{aligned}
$$

It will also imply that

$$
\begin{aligned}
&\frac{(2 + (\pi_B^-(l_i))^2 + (\pi_B^+(l_i))^2 - |(\mu_B^-(l_i))^2 - (\nu_B^-(l_i))^2| - |(\mu_B^+(l_i))^2 - (\nu_B^+(l_i))^2|)}{2 + (\pi_B^-(l_i))^2 + (\pi_B^+(l_i))^2} \\
&\leqslant \frac{(2 + (\pi_N^-(l_i))^2 + (\pi_N^+(l_i))^2 - |(\mu_N^-(l_i))^2 - (\nu_N^-(l_i))^2| - |(\mu_N^+(l_i))^2 - (\nu_N^+(l_i))^2|)}{2 + (\pi_N^-(l_i))^2 + (\pi_N^+(l_i))^2}, \\
&\quad \forall l_i \in L.
\end{aligned}
$$

Thus, $E(B) \leqslant E(N)$. Similarly, for $B \supseteq N$ with $\mu_N^-(l_i) \geqslant \nu_N^-(l_i)$ and $\mu_N^+(l_i) \geqslant \nu_N^+(l_i)$, for each $l_i \in L$, it is demonstrated that $E(B) \leqslant E(N)$. □

In the following, the developed IVPF-entropy is evaluated against existing IVPF-entropy measures to reveal its effectiveness (Peng and Li, 2019; Mishra *et al.*, 2023; Rani *et al.*, 2023; Moazzamigodarzi *et al.*, 2025). Table S1 (Supplementary Material) reviews the formulations of these measures.

Example 1 (Hung and Yang (2006)). Consider an IVPFS $B = \{(l_i, ([\mu_B^-(l_i), \mu_B^+(l_i)], [\nu_B^-(l_i), \nu_B^+(l_i)])) : l_i \in L\}$. For any $m \in \mathbb{R}^+$, define the IVPFS B^m as follows:

$$
B^m = \left\{\left(l_i, \left(\left[\left(\mu^-(l_i)\right)^m, \left(\mu^+(l_i)\right)^m\right], \left[\left(\nu^-(l_i)\right)^m, \left(\nu^+(l_i)\right)^m\right]\right)\right) : l_i \in L\right\}. \tag{9}
$$

Table 1
Evaluation of the proposed and existing entropy measures for IVPFSs.

	E_1 (.)	E_2 (.)	E_3 (.)	E_4 (.)	E_5 (.)	E (.)
B	0.478	0.489	0.524	0.508	0.637	0.579
B^2	0.424	0.433	0.472	0.364	0.593	0.532
B^3	0.380	0.394	0.424	0.254	0.519	0.473
B^4	0.339	0.356	0.382	0.191	0.464	0.423

The IVPFS B is denoted on the universal set $L = \{6, 7, 8, 9, 10\}$:

$$B = \left\{ \begin{array}{l} \langle 6, [0.1, 0.2], [0.6, 0.7]\rangle, \langle 7, [0.3, 0.5], [0.4, 0.5]\rangle, \\ \langle 8, [0.6, 0.7], [0.1, 0.2]\rangle, \langle 9, [0.8, 0.9], [0, 0.1]\rangle, \langle 10, [1, 1], [0, 0]\rangle \end{array} \right\}.$$

Similarly, we can define the following IVPFSs:

$$B^2 = \left\{ \begin{array}{l} \langle 6, [0.01, 0.04], [0.84, 0.91]\rangle, \langle 7, [0.09, 0.25], [0.64, 0.75]\rangle, \\ \langle 8, [0.36, 0.49], [0.19, 0.36]\rangle, \langle 9, [0.64, 0.81], [0, 0.19]\rangle, \langle 10, [1, 1], [0, 0]\rangle \end{array} \right\},$$

$$B^3 = \left\{ \begin{array}{l} \langle 6, [0.0010, 0.0080], [0.9360, 0.9730]\rangle, \langle 7, [0.0270, 0.1250], [0.7840, 0.8750]\rangle, \\ \langle 8, [0.2160, 0.3430], [0.2710, 0.4880]\rangle, \langle 9, [0.5120, 0.7290], [0, 0.2710]\rangle, \\ \langle 10, [1, 1], [0, 0]\rangle \end{array} \right\},$$

$$B^4 = \left\{ \begin{array}{l} \langle 6, [0.0001, 0.0016], [0.9744, 0.9919]\rangle, \langle 7, [0.0081, 0.0625], [0.8704, 0.9375]\rangle, \\ \langle 8, [0.1296, 0.2401], [0.3439, 0.5904]\rangle, \langle 9, [0.4096, 0.6561], [0, 0.3439]\rangle, \\ \langle 10, [1, 1], [0, 0]\rangle \end{array} \right\}.$$

To capture the semantics of linguistic expressions, IVPFSs are adopted to model the effectiveness of structural linguistic variables. In line with the classification framework introduced by De *et al.* (2000), the linguistic term "LARGE" serves as the baseline reference. Based on this foundation, a hierarchy of modified linguistic intensifiers is defined, including "More or Less LARGE" (B), "Very LARGE" (B^2), "Quite Very LARGE" (B^3), and "Very Very LARGE" (B^4), to represent increasing degrees of emphasis.

In accordance with the underlying mathematical properties, the entropy measures defined for interval-valued Pythagorean fuzzy sets must satisfy the criteria established by Hung and Yang (2006), as summarized in Table 1; i.e. $E(B) > E(B^2) > E(B^3) > E(B^4)$. It is evident that the proposed entropy measure demonstrates sound behaviour and compares favourably with existing interval-valued Pythagorean fuzzy entropy measures.

3.3. *An Improved Score Function for IVPFN*

The notion of a score function was proposed by Peng and Yang (2016), which can derive a crisp value directly from a single IVPFV. Further, the researchers have paid considerable emphasis on studying the score function and its properties for comparing the IVPFVs. However, there are some situations where existing IVPF-score functions (e.g. Alrasheedi and Jeevaraj, 2023; Rani *et al.*, 2023) produce the same score values. To overcome the limitations of existing IVPF-score functions, a modified score function for an IVPFV is offered.

Let $\delta = ([\mu^-, \mu^+], [\nu^-, \nu^+])$ be an IVPFN. Then a modified score function is proposed for 'δ' and given as

$$S(\delta) = \{(\mu^-)^2 + (\mu^+)^2 + (1 - (\nu^-)^2) + (1 - (\nu^+)^2) + (\mu^-)^2(\mu^+)^2 - \sqrt{(1 - (\nu^-)^2)(1 - (\nu^+)^2)}\}/4. \tag{10}$$

Property 1. For an IVPFN $\delta = ([\mu^-, \mu^+], [\nu^-, \nu^+])$, $S(\delta)$ is continuously increasing as a function of μ^- and μ^+, and continuously diminishing over ν^- and ν^+.

Proof. We differentiate Eq. (10) partially to μ^- and μ^+, respectively:

$$\frac{\partial S(\delta)}{\partial \mu^-} = \frac{\mu^-}{2}(1 + (\mu^+)^2) \quad \text{and} \quad \frac{\partial S(\delta)}{\partial \mu^+} = \frac{\mu^+}{2}(1 + (\mu^-)^2).$$

Similarly, we differentiate Eq. (10) partially to ν^- and ν^+, respectively:

$$\frac{\partial S(\delta)}{\partial \nu^-} = -\frac{\nu^-}{2}\left(1 - \frac{(1 - (\nu^+)^2)}{\sqrt{(1 - (\nu^-)^2)(1 - (\nu^-)^2)}}\right) \quad \text{and}$$
$$\frac{\partial S(\delta)}{\partial \nu^+} = -\frac{\nu^+}{2}\left(1 - \frac{(1 - (\nu^-)^2)}{\sqrt{(1 - (\nu^-)^2)(1 - (\nu^-)^2)}}\right).$$

Since $0 \leqslant \mu^- \leqslant \mu^+ \leqslant 1$ and $0 \leqslant \nu^- \leqslant \nu^+ \leqslant 1$, therefore, $\frac{\partial S(\delta)}{\partial \mu^-} \geqslant 0$, $\frac{\partial S(\delta)}{\partial \mu^+} \geqslant 0$, $\frac{\partial S(\delta)}{\partial \nu^-} \leqslant 0$ and $\frac{\partial S(\delta)}{\partial \nu^+} \leqslant 0$. Consequently, we obtain that the score function $S(\delta)$ raises continuously over μ^- and μ^+, and decreases continuously over ν^- and ν^+. □

Property 2. The suggested score function '$S(\delta)$' of an IVPFN $\delta = ([\mu^-, \mu^+], [\nu^-, \nu^+])$ satisfies $S([1, 1], [0, 0]) = 1$ and $S([0, 0], [1, 1]) = 0$.

Proof. According to Property 1, we obtain that the score function $S(\delta)$ raises continuously over μ^- and μ^+, and decreases continuously over ν^- and ν^+. Therefore, $S(\delta)$ can have the maximum value '1' if and only if $\delta = ([1, 1], [0, 0])$ and $S(\delta)$ can have the minimum value '0' if and only if $\delta = ([0, 0], [1, 1])$. □

Property 3. Let $\delta_1 = ([\mu_1^-, \mu_1^+], [\nu_1^-, \nu_1^+])$ and $\delta_2 = ([\mu_2^-, \mu_2^+], [\nu_2^-, \nu_2^+])$ be two IVPFNs. If $\mu_1^- > \mu_2^-$, $\mu_1^+ > \mu_2^+$, $\nu_1^- < \nu_2^-$ and $\nu_1^+ < \nu_2^+$, then $S(\delta_1) > S(\delta_2)$.

Proof. From Property 1, we obtain that the score function $S(\delta)$ increases continuously over μ^- and μ^+, and decreases continuously over ν^- and ν^+. Hence, if $\mu_1^- > \mu_2^-$, $\mu_1^+ > \mu_2^+$, $\nu_1^- < \nu_2^-$ and $\nu_1^+ < \nu_2^+$, then $S(\delta_1) > S(\delta_2)$. □

To appraise the feasibility of the developed score function $S(\delta)$ for comparing IVPFNs, Table 2 presents a comparison between the results achieved using the proposed score function and those obtained from the score functions developed by Peng and Yang (2016), Garg

(2016), Garg (2017), Peng and Li (2019), Alrasheedi and Jeevaraj (2023) and Rani *et al.* (2023).

I. To compare three different IVPFNs $\delta_1 = ([0.4, 0.7], [0.4, 0.7])$, $\delta_2 = ([0.3, 0.4], [0.3, 0.4])$, and $\delta_8 = ([0, 0.7], [0, 0.7])$, the score function by Peng and Yang (2016) presents the same result, '0'. Additionally, Peng & Yang's score function is not able to distinguish the IVPFNs $\delta_3 = ([0, 0.5], [0.1, 0.7])$ and $\delta_4 = ([0.3, 0.4], [0.5, 0.5])$ as $S_1(\delta_3) = -0.125 = S_1(\delta_4)$.

II. Similarly, Garg's score formula (Garg, 2016) is unable to compare two different IVPFNs, $\delta_5 = ([0, 1], [0, 0])$ and $\delta_6 = ([0, 0], [1, 1])$, as $S_2(\delta_5) = 0 = S_2(\delta_6)$.

III. For $\delta_1 = ([0.4, 0.7], [0.4, 0.7])$ and $\delta_2 = ([0.3, 0.4], [0.3, 0.4])$, we get $S_3(\delta_1) = 0 = S_3(\delta_2)$, indicating that Garg's score formula $S_3(.)$ (Garg, 2017) cannot distinguish δ_1 and δ_2.

IV. The score function is used to convert IVPFN into a crisp number, but the score formula $S_4(.)$ offered by Peng and Li (2019) is unable to convert an IVPFN $\delta_6 = ([0, 0], [1, 1])$ into a crisp value and provides an unreasonable result; i.e. "division-by-zero problem". For the same IVPFN $\delta_6 = ([0, 0], [1, 1])$, the result obtained by the score function offered by Alrasheedi and Jeevaraj (2023) is not consistent with the other score results.

V. For any two IVPFNs $\delta_3 = ([0, 0.5], [0.1, 0.7])$ and $\delta_7 = ([0.7, 0.7], [\sqrt{0.495}, \sqrt{0.51}])$, we apply the score function $S_6(.)$ and obtain the result, as $S_6(\delta_3) = 0.124 = S_6(\delta_7)$, it means that the score function offered by Rani *et al.* (2023) fails to distinguish these IVPFNs.

According to Table 2, it can be observed that the suggested score function $S(\delta)$ can efficiently avoid the inadequacies of existing IVPF-score functions (Peng and Yang, 2016; Garg, 2016; 2017; Peng and Li, 2019; Alrasheedi and Jeevaraj, 2023; Rani *et al.*, 2023). The proposed score function is capable of distinguishing between two IVPFNs even in cases where the existing score functions fail to operate.

3.4. *Proposed Distance Measure for IVPFSs*

The distance measure concept is a mathematical term that calculates the dissimilarity score between two objects/entities.

Theorem 2. *For $B, N \in IVPFSs(L)$, the function 'd', defined by*

$$d(B,N) = \frac{1}{r}\sum_{i=1}^{r}\sqrt{\left[\left(\frac{(\mu_B^-(l_i))^2+(\mu_B^+(l_i))^2-(\nu_B^-(l_i))^2-(\nu_B^+(l_i))^2}{4} - \frac{(\mu_N^-(l_i))^2+(\mu_N^+(l_i))^2-(\nu_N^-(l_i))^2-(\nu_N^+(l_i))^2}{4}\right)^2 + \frac{1}{3}\left(\frac{(\mu_B^-(l_i))^2+(\mu_B^+(l_i))^2+(\nu_B^-(l_i))^2+(\nu_B^+(l_i))^2}{4} - \frac{(\mu_N^-(l_i))^2+(\mu_N^+(l_i))^2+(\nu_N^-(l_i))^2+(\nu_N^+(l_i))^2}{4}\right)^2\right]} \tag{11}$$

is a distance measure for IVPFSs.

Table 2
Comparative results for the IVPF-score function.

Score formulae	$\delta_1 =$ ([0.4, 0.7], [0.4, 0.7])	$\delta_2 =$ ([0.3, 0.4], [0.3, 0.4])	$\delta_3 =$ ([0, 0.5], [0.1, 0.7])	$\delta_4 =$ ([0.3, 0.4], [0.5, 0.5])	$\delta_5 =$ ([0, 1], [0, 0])	$\delta_6 =$ ([0, 0], [1, 1])	$\delta_7 =$ ([0.7, 0.7], [$\sqrt{0.495}$, $\sqrt{0.51}$])	$\delta_8 =$ ([0, 0.7], [0, 0.7])
Peng and Yang (2016) $S_1(\delta) = \frac{1}{2}((\mu^-)^2 + (\mu^+)^2 - (\nu^-)^2 - (\nu^+)^2)$	0*	0	−0.125	−0.125	0.5	−1	−0.013	0
Garg (2016) $S_2(\delta) = \frac{((\mu^-)^2-\sqrt{1-(\mu^-)^2-(\nu^-)^2})+((\mu^+)^2-\sqrt{1-(\mu^+)^2-(\nu^+)^2})}{2}$	−0.158	−0.74	−0.627	−0.665	0	0	0.429	−0.326
Garg (2017) $S_3(\delta) = \frac{1}{2}\left(\begin{array}{l}((\mu^-)^2 - (\nu^-)^2)(1 + \sqrt{1 - (\mu^-)^2 - (\nu^-)^2}) \\ +((\mu^-)^2 - (\nu^-)^2)(1 + \sqrt{1 - (\mu^+)^2 - (\nu^+)^2})\end{array}\right)$	0	0	−0.191	−0.225	0.5	−1	−0.013	0
Peng and Li (2019) $S_4(\delta) = \frac{((\mu^-)^2+(\mu^+)^2-(\nu^-)^2-(\nu^+)^2)}{2} + \frac{((\mu^-)^2-(\mu^-)^2(\nu^+)^2-(\mu^-)^4)}{2((\nu^-)^2-(\mu^+)^2(\nu^-)^2-(\nu^-)^4+1)}$	0.052	0.035	−0.125	−0.099	0.5	NaN**	−0.009	0
Alrasheedi and Jeevaraj (2023) $S_5(\delta) = \frac{1}{3}((\mu^-)^2 + (\mu^+)^2 - (\nu^-)^2 - (\nu^+)^2 + (\mu^-)^2(\mu^+)^2 + (\nu^-)^2(\nu^+)^2)$	0.052	0.01	−0.082	−0.058	0.333	−0.333	0.156	0
Rani *et al.* (2023) $S_6(\delta) = \frac{(\mu^-)^2(1-(\nu^+)^2)+(\mu^+)^2(1-(\nu^-)^2)-(\mu^-)^2(\mu^+)^2}{2}$	0.207	0.103	0.124	0.087	0.5	0	0.124	0.245
Proposed IVPF-score function	0.356	0.285	0.26	0.254	0.5	0	0.429	0.321

*Bold denotes unreasonable results; ** NaN: Not a number.

Proof. To prove this theorem, we need to prove the axioms (p_1)–(p_4) of Definition 6.

(b1) For any two IVPFSs P and Q, we have $0 \leqslant \mu_B^-(l_i) \leqslant \mu_B^+(l_i) \leqslant 1, 0 \leqslant \mu_N^-(l_i) \leqslant \mu_N^+(l_i) \leqslant 1, 0 \leqslant \nu_B^-(l_i) \leqslant \nu_B^+(l_i) \leqslant 1$ and $0 \leqslant \nu_N^-(l_i) \leqslant \nu_N^+(l_i) \leqslant 1, \forall l_i \in L$. Here, we can observe that each term of Eq. (11) is non-negative; and then $d(B, N) \geqslant 0$. Moreover, for any two IVPFSs $B = ([1, 1], [0, 0])$ and $N = ([0, 0], [1, 1])$ in the fixed universal set L, then we get $d(B, N) \leqslant 1$.

(b2) Suppose that $d(B, N) = 0$, and based on Eq. (11):

$$\begin{pmatrix} \frac{(\mu_B^-(l_i))^2+(\mu_B^+(l_i))^2-(\nu_B^-(l_i))^2-(\nu_B^+(l_i))^2}{4} \\ -\frac{(\mu_N^-(l_i))^2+(\mu_N^+(l_i))^2-(\nu_N^-(l_i))^2-(\nu_N^+(l_i))^2}{4} \end{pmatrix}^2 = 0, \tag{12}$$

and

$$\begin{pmatrix} \frac{(\mu_B^-(l_i))^2+(\mu_B^+(l_i))^2+(\nu_B^-(l_i))^2+(\nu_B^+(l_i))^2}{4} \\ -\frac{(\mu_N^-(l_i))^2+(\mu_N^+(l_i))^2+(\nu_N^-(l_i))^2+(\nu_N^+(l_i))^2}{4} \end{pmatrix}^2 = 0. \tag{13}$$

Solving Eqs. (12) and (13), we get $\mu_B^-(l_i) = \mu_N^-(l_i), \mu_B^+(l_i) = \mu_N^+(l_i), \nu_B^-(l_i) = \nu_N^-(l_i)$ and $\nu_B^+(l_i) = \nu_N^+(l_i), \forall l_i \in L$. Hence, $B = N$.

On the contrary, consider $B = N$, then $\mu_B^-(l_i) = \mu_N^-(l_i), \mu_B^+(l_i) = \mu_N^+(l_i), \nu_B^-(l_i) = \nu_N^-(l_i)$ and $\nu_B^+(l_i) = \nu_N^+(l_i), \forall l_i \in L$. Then, from Eq. (11), it is obvious that $d(B, N) = 0$.

(b3) Since the result follows directly from Eq. (11), the proof is omitted.

(b4) For any three IVPFSs B, N and G, let $B \subseteq N \subseteq G$, we have $\mu_B^-(l_i) \leqslant \mu_N^-(l_i) \leqslant \mu_G^-(l_i), \mu_B^+(l_i) \leqslant \mu_N^+(l_i) \leqslant \mu_G^+(l_i), \nu_G^-(l_i) \leqslant \nu_N^-(l_i) \leqslant \nu_B^-(l_i)$ and $\nu_G^+(l_i) \leqslant \nu_N^+(l_i) \leqslant \nu_B^+(l_i), \forall l_i \in L$.

Consider a function $f(z_1, z_2, z_3, z_4)$, where

$$f(z_1, z_2, z_3, z_4) = \begin{pmatrix} \left(\left(z_1^2 + z_2^2 - z_3^2 - z_4^2\right) - \left(\alpha_1^2 + \alpha_2^2 - \alpha_3^2 - \alpha_4^2\right)\right)^2 \\ +\frac{1}{3}\left(\left(z_1^2 + z_2^2 + z_3^2 + z_4^2\right) - \left(\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2\right)\right)^2 \end{pmatrix}, \tag{14}$$

where $0 \leqslant z_1, z_2, z_3, z_4 \leqslant 1$ and $0 \leqslant \alpha_1, \alpha_2, \alpha_3, \alpha_4 \leqslant 1$.

Differentiate $f(z_1, z_2, z_3, z_4)$ partially with respect to z_1, z_2, z_3 and z_4, respectively, we have

$$\frac{\partial f(z_1, z_2, z_3, z_4)}{\partial z_1} = \frac{8}{3} z_1 \left(2\left(z_1^2 + z_2^2\right) - \left(z_3^2 + z_4^2\right) - 2\left(\alpha_1^2 + \alpha_2^2\right) + \left(\alpha_3^2 + \alpha_4^2\right)\right), \tag{15}$$

$$\frac{\partial f(z_1, z_2, z_3, z_4)}{\partial z_2} = \frac{8}{3} z_2 \left(2\left(z_1^2 + z_2^2\right) - \left(z_3^2 + z_4^2\right) - 2\left(\alpha_1^2 + \alpha_2^2\right) + \left(\alpha_3^2 + \alpha_4^2\right)\right), \tag{16}$$

$$\frac{\partial f(z_1, z_2, z_3, z_4)}{\partial z_3} = -\frac{8}{3} z_3 \big((z_1^2 + z_2^2) - 2(z_3^2 + z_4^2) - (\alpha_1^2 + \alpha_2^2) + 2(\alpha_3^2 + \alpha_4^2) \big), \tag{17}$$

$$\frac{\partial f(z_1, z_2, z_3, z_4)}{\partial z_4} = -\frac{8}{3} z_4 \big((z_1^2 + z_2^2) - 2(z_3^2 + z_4^2) - (\alpha_1^2 + \alpha_2^2) + 2(\alpha_3^2 + \alpha_4^2) \big). \tag{18}$$

For the conditions $0 \leqslant \alpha_1 \leqslant z_1 \leqslant 1$, $0 \leqslant \alpha_2 \leqslant z_2 \leqslant 1$, $0 \leqslant z_3 \leqslant \alpha_3 \leqslant 1$ and $0 \leqslant z_4 \leqslant \alpha_4 \leqslant 1$, we have $\frac{\partial f(z_1,z_2,z_3,z_4)}{\partial z_1} \geqslant 0$, $\frac{\partial f(z_1,z_2,z_3,z_4)}{\partial z_2} \geqslant 0$, $\frac{\partial f(z_1,z_2,z_3,z_4)}{\partial z_3} \leqslant 0$ and $\frac{\partial f(z_1,z_2,z_3,z_4)}{\partial z_4} \leqslant 0$, respectively. Here, we can observe that the function $f(z_1, z_2, z_3, z_4)$ is increasing with respect to z_1 and z_2 for $0 \leqslant \alpha_1 \leqslant z_1 \leqslant 1$ and $0 \leqslant \alpha_2 \leqslant z_2 \leqslant 1$, respectively, and decreasing with respect to z_3 and z_4 for $0 \leqslant z_3 \leqslant \alpha_3 \leqslant 1$ and $0 \leqslant z_4 \leqslant \alpha_4 \leqslant 1$, respectively. Now, let us assume three IVPFSs B, N and G such that $B \subseteq N \subseteq G$, then $\mu_B^-(l_i) \leqslant \mu_N^-(l_i) \leqslant \mu_G^-(l_i)$, $\mu_B^+(l_i) \leqslant \mu_N^+(l_i) \leqslant \mu_G^+(l_i)$, $\nu_G^-(l_i) \leqslant \nu_N^-(l_i) \leqslant \nu_B^-(l_i)$ and $\nu_G^+(l_i) \leqslant \nu_N^+(l_i) \leqslant \nu_B^+(l_i)$, $\forall l_i \in L$. From the monotonicity of $f(z_1, z_2, z_3, z_4)$, we can have $d(B, G) \geqslant d(B, N)$. By a similar argument, we can demonstrate that $d(B, G) \geqslant d(N, G)$.

Theoretically, it is clear that the proposed function, given in Eq. (11), satisfies all the required characteristics of a distance measure under the IVPF environment. To prove the superiority of the introduced measure, a comparison with several existing IVPF distance measures is made. To this aim, we first recall these IVPF distance measures, given as follows:

- **IVPF distance by Zhang (2016)**

$$d_1(B, N) = \frac{1}{4r} \sum_{i=1}^{r} \begin{pmatrix} |(\mu_B^-(l_i))^2 - (\mu_N^-(l_i))^2| + |(\mu_B^+(l_i))^2 - (\mu_N^+(l_i))^2| \\ +|(\nu_B^-(l_i))^2 - (\nu_N^-(l_i))^2| + |(\nu_B^+(l_i))^2 - (\nu_N^+(l_i))^2| \\ +|(\pi_B^-(l_i))^2 - (\pi_N^-(l_i))^2| + |(\pi_B^+(l_i))^2 - (\pi_N^+(l_i))^2| \end{pmatrix}. \tag{19}$$

- **IVPF distance by Kumar *et al.* (2020)**

$$d_2(B, N) = \sqrt[p]{\frac{1}{4r} \sum_{i=1}^{r} \begin{pmatrix} |(\mu_B^-(l_i))^2 - (\mu_N^-(l_i))^2|^p + |(\mu_B^+(l_i))^2 - (\mu_N^+(l_i))^2|^p \\ +|(\nu_B^-(l_i))^2 - (\nu_N^-(l_i))^2|^p + |(\nu_B^+(l_i))^2 - (\nu_N^+(l_i))^2|^p \\ +|(\pi_B^-(l_i))^2 - (\pi_N^-(l_i))^2|^p + |(\pi_B^+(l_i))^2 - (\pi_N^+(l_i))^2|^p \end{pmatrix}}. \tag{20}$$

- **IVPF distance by Liu *et al.* (2025)**

$$d_3(B, N) = \sqrt{\frac{1}{4r} \sum_{i=1}^{r} \begin{pmatrix} \frac{((\mu_B^-(l_i))^2 - (\mu_N^-(l_i))^2)^2}{(\mu_B^-(l_i))^2 + (\mu_N^-(l_i))^2} + \frac{((\mu_B^+(l_i))^2 - (\mu_N^+(l_i))^2)^2}{(\mu_B^+(l_i))^2 + (\mu_N^+(l_i))^2} \\ + \frac{((\nu_B^-(l_i))^2 - (\nu_N^-(l_i))^2)^2}{(\nu_B^-(l_i))^2 + (\nu_N^-(l_i))^2} + \frac{((\nu_B^+(l_i))^2 - (\nu_N^+(l_i))^2)^2}{(\nu_B^+(l_i))^2 + (\nu_N^+(l_i))^2} \end{pmatrix}}. \tag{21}$$

Table 3
Comparison of outcomes of developed and existing IVPF distance measures.

	Set-1	Set-2	Set-3	Set-4	Set-5	Set-6
B	([0.25, 0.35], [0.25, 0.35]),	([0.25, 0.35], [0.35, 0.45])	([1, 1], [0, 0])	([0.3, 0.3], [0.3, 0.3])	([0.1, 0.1], [0.2, 0.2])	([1, 1], [0, 0])
N	([0.35, 0.45], [0.35, 0.45])	([0.35, 0.45], [0.25, 0.35])	([0, 0], [0, 0])	([0, 0], [0, 0])	([0.3, 0.3], [0.4, 0.4])	([0, 0], [1, 1])
$d_1(B, N)$	0.14	0.07	**1***	0.18	0.2	**1**
$d_2(B, N)$	**0.01**	**0.01**	0.8409	0.09	0.04	1
$d_3(B, N)$	0.56	0.28	**4**	0.72	0.8	**4**
$d_4(B, N)$	**0.052**	**0.052**	0.729	0.066	0.077	1
$d_5(B, N)$	**0.0038**	**0.0038**	0.5	0.0055	0.0075	1
$d_6(B, N)$	0.14	0.07	**1**	0.18	0.2	**1**
Proposed IVPF distance measure	0.0404	0.07	0.5774	0.052	0.0611	1

*Bold denotes unreasonable results.

- **IVPF distance by Yu *et al.* (2025)**

$$d_4(B,N) = 1 - \log_2\left(2 - \sqrt[q]{\frac{1}{4r}\sum_{i=1}^{r}\begin{pmatrix}|(\mu_B^-(l_i))^2 - (\mu_N^-(l_i))^2|^q + |(\mu_B^+(l_i))^2 - (\mu_N^+(l_i))^2|^q \\ +|(\nu_B^-(l_i))^2 - (\nu_N^-(l_i))^2|^q + |(\nu_B^+(l_i))^2 - (\nu_N^+(l_i))^2|^q\end{pmatrix}}\right), \tag{22}$$

$$d_5(B,N) = 1 - \log_2\left(2 - \sqrt[q]{\frac{1}{4r}\sum_{i=1}^{r}\begin{pmatrix}|(\mu_B^-(l_i))^2 - (\mu_N^-(l_i))^2|^q + |(\mu_B^+(l_i))^2 - (\mu_N^+(l_i))^2|^q \\ +|(\nu_B^-(l_i))^2 - (\nu_N^-(l_i))^2|^q + |(\nu_B^+(l_i))^2 - (\nu_N^+(l_i))^2|^q \\ +|(\pi_B^-(l_i))^2 - (\pi_N^-(l_i))^2|^q + |(\pi_B^+(l_i))^2 - (\pi_N^+(l_i))^2|^q\end{pmatrix}}\right). \tag{23}$$

- **IVPF distance by Moazzamigodarzi *et al.* (2025)**

$$\begin{aligned} d_6(B,N) = \frac{1}{4r(e^2-1)}\sum_{i=1}^{r}\Bigg[&\left((\mu_B^-(l_i))^2 - (\mu_N^-(l_i))^2\right)\left(e^{\left(\frac{4(\mu_B^-(l_i))^2}{1+(\mu_B^-(l_i))^2+(\mu_N^-(l_i))^2}\right)} - e^{\left(\frac{4(\mu_N^-(l_i))^2}{1+(\mu_B^-(l_i))^2+(\mu_N^-(l_i))^2}\right)}\right) \\ &+\left((\mu_B^+(l_i))^2 - (\mu_N^+(l_i))^2\right)\left(e^{\left(\frac{4(\mu_B^+(l_i))^2}{1+(\mu_B^+(l_i))^2+(\mu_N^+(l_i))^2}\right)} - e^{\left(\frac{4(\mu_N^+(l_i))^2}{1+(\mu_B^+(l_i))^2+(\mu_N^+(l_i))^2}\right)}\right) \\ &+\left((\nu_B^-(l_i))^2 - (\nu_N^-(l_i))^2\right)\left(e^{\left(\frac{4(\nu_B^-(l_i))^2}{1+(\nu_B^-(l_i))^2+(\nu_N^-(l_i))^2}\right)} - e^{\left(\frac{4(\nu_N^-(l_i))^2}{1+(\nu_B^-(l_i))^2+(\nu_N^-(l_i))^2}\right)}\right) \\ &+\left((\nu_B^+(l_i))^2 - (\nu_N^+(l_i))^2\right)\left(e^{\left(\frac{4(\nu_B^+(l_i))^2}{1+(\nu_B^+(l_i))^2+(\nu_N^+(l_i))^2}\right)} - e^{\left(\frac{4(\nu_N^+(l_i))^2}{1+(\nu_B^+(l_i))^2+(\nu_N^+(l_i))^2}\right)}\right)\Bigg]. \end{aligned} \tag{24}$$

Table 3 presents the required comparative outcomes of suggested and existing measures. It is observed that the previously developed IVPF distances: i.e. $d_2(B, N)$, $d_4(B, N)$

and $d_5(B, N)$, produced the same results for two various sets of IVPFSs, given by Set-1 and Set-2. Similarly, the distance formulae $d_1(B, N)$, $d_3(B, N)$ and $d_6(B, N)$ failed to distinguish the two sets of IVPFSs, given by Set-3 and Set-6. In addition, the property of the distance measure in Theorem 2 is not satisfied using $d_3(B, N)$ in Case 6. As $B = ([1, 1], [0, 0])$ and $N = ([0, 0], [1, 1])$ are the maximum and the minimum IVPFNs, therefore, it is expected that the distance between B and N is the maximum distance, arriving at '1', while the IVPF distance suggested by Liu *et al.* (2025) obtains the result as '4'. The proposed IVPF distance does not exhibit any counter-intuitive behaviour and demonstrates a strong performance compared to others. □

4. The IVPF-WENSLO-RANCOM-CRADIS Model

With the help of proposed measures in Section 3, this section develops a novel fuzzy integrated decision support framework under the setting of IVPFSs and named "IVPF-WENSLO-RANCOM-CRADIS". Figure 1 presents the flowchart that illustrates the execution steps of the developed framework.

4.1. *Aggregate the Individual Decision Matrices*

This procedure involves the following steps:

1. Create the Linguistic Assessment Decision matrix (LADM) on the basis of the opinions of a set of experts.
2. Determine the significance value of each expert.
3. Utilize the experts' weights in the IVPFWA operator and find the aggregated opinion about the performance of each alternative over each criterion.

4.2. *Integrated Weighting Model for Criteria Weights Determination*

Let $w = (w_1, w_2, \ldots, w_t)^T$ be the weight vector of the criteria set $V = \{v_1, v_2, \ldots, v_t\}$ with $w_j \in [0, 1]$ and $\sum_{j=1}^{t} w_j = 1$. To specify the numeric value of w, we incorporate the IVPF-WENSLO and the IVPF-RANCOM models for objective and subjective weights of criteria, on the basis of the following steps:

1. Obtain the objective weight of each criterion through the IVPF-WENSLO model.
2. Identify the subjective weight of each criterion using the IVPF-RANCOM method.
3. Integrate the objective and subjective weighting to determine the combined weight of each criterion.

4.3. *Integrated CRADIS Model for Alternatives' Ranking*

The suggested CRADIS model is implemented through the following steps:

1. Find the IVPF ideal and IVPF anti-ideal solutions.

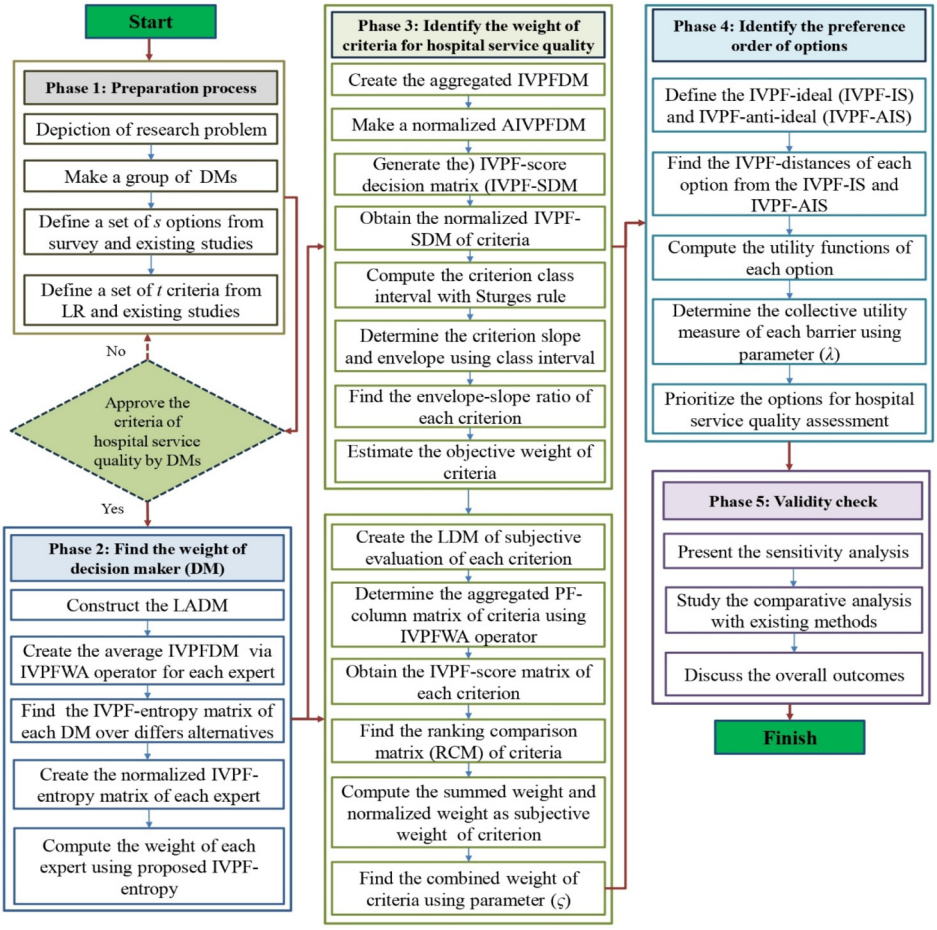


Fig. 1. Representation of proposed decision-making methodology.

2. Obtain the distances from each alternative to the IVPF ideal and anti-ideal solutions.
3. Determine the utility functions.
4. Find the collective measure of each alternative.
5. Rank the alternatives as per the values of the collective measure.

5. Case Study: Hospital Service Quality Assessment

To demonstrate the applicability of the proposed method, it was applied to assess the service quality of five hospitals in Patna district, Bihar, India. Patna serves as the capital and largest city of Bihar. It is also the 19th most populous city in the country and caters to a very large population. Since the management of the considered hospitals did not agree to reveal their identity, we referred to the name of the hospitals as Hospital-1 (A_1), Hospi-

Stage 1: Process for aggregating the individual decision opinions

No. of steps	Description of the procedure
Step 1.	**Create the LADM** Suppose that a MCGDM problem involves a finite set of 'r' alternatives $A = \{A_1, A_2, \ldots, A_s\}$ and they are to be ranked over a set of 't' criteria $V = \{v_1, v_2, \ldots, v_t\}$. Let us invite a set of 'q' DMs $G = \{g_1, g_2, \ldots, g_q\}$ to give the linguistic assessment ratings of alternatives concerning each criterion. Consequently, a LADM is formed as $Y = (Y_{ij}^{(k)})_{s\times t}$, where $Y_{ij}^{(k)}$ stands for the linguistic assessment rating of ith alternative over jth criterion, provided by kth DM. Using the Likert's scale, the LADM is transformed into IVPF decision matrix (IVPFDM) $F = (\upsilon_{ij}^{(k)})_{s\times t}$.
Step 2.	**Derive the DMs' significance values** Let $\omega = (\omega_1, \omega_2, \ldots, \omega_q)^T$ be the weight vector of q DMs $G = \{g_1, g_2, \ldots, g_q\}$ with $0 \leqslant \omega_j \leqslant 1$ and $\omega_1 + \omega_2 + \cdots + \omega_q = 1$. To find the numeric value of ω, we first create the average IVPFDM $\bar{F} = (\bar{\upsilon}_{ij}^{(k)})_{s\times q}$ using IVPFWA operator for each expert and then find the entropy of each DE over different alternatives through proposed entropy (6). Finally, the formula for finding ω_k is given as $$\omega_k = \frac{1 - \sum_{i=1}^{s}(\bar{E}(\upsilon_{ik}))}{q - \sum_{k=1}^{q}\sum_{i=1}^{s}(\bar{E}(\upsilon_{ik}))} \quad (k = 1, 2, \ldots, q), \tag{25}$$ where $\bar{E}(\upsilon_{ik})$ displays the normalized entropy of kth DM, given as $\bar{E}(\upsilon_{ik}) = \frac{E(\upsilon_{ik})}{\max_i E(\upsilon_{ik})}$ $(k = 1, 2, \ldots, q)$.
Step 3.	**Create the IVPF-Aggregated Decision matrix (IVPFADM)** It is important to note that, in the development of IVPFADM, individual viewpoints must be aggregated into a collective opinion to facilitate group decision-making. To construct an IVPFADM $F_A = (\tilde{\upsilon}_{ij})_{s\times t}$, the individual opinions of DMs about each alternative are aggregated through the use of experts' weights into the IVPFWA operator, where $\tilde{\upsilon}_{ij}$ is an IVPFN, denotes an assessment value of ith alternative by means of jth criterion $$\tilde{\upsilon}_{ij} = \left(\left[1 - \prod_{k=1}^{q}(1 - (\mu_{\upsilon_{ij}^k}^{-})^2)^{\omega_k}, 1 - \prod_{k=1}^{q}(1 - (\mu_{\upsilon_{ij}^k}^{+})^2)^{\omega_k}\right], \left[\prod_{k=1}^{q}(\nu_{\upsilon_{ij}^k}^{-})^{2\omega_k}, \prod_{ik=1}^{q}(\nu_{\upsilon_{ij}^k}^{+})^{2\omega_k}\right]\right). \tag{26}$$

tal-2 (A_2), Hospital-3 (A_3), Hospital-4 (A_4), and Hospital-5 (A_5). In order to determine the criteria, a comprehensive literature survey was conducted with the keywords "healthcare service quality", "patient satisfaction", "patient-perceived", and "multi-criteria decision-making". Table 4 presents the list of selected criteria with their sources. Next, a group of four experts is asked to confirm the validity of selected service quality dimensions and criteria for evaluating the hospitals. This group includes experts from healthcare administration, hospital staff from the emergency department, ward service provider, and quality consultants, and details of DEs are presented in Table 5. In the following, we present the step-by-step procedure for assessing the quality of service at the five hospitals of Patna.

Stage 1: Process for aggregating the individual decision opinions

***Step* 1:** With the use of Table S2, a group of four experts is provided with their linguistic opinions about the performance of each hospital in respect of each evaluation criterion. Thus, a linguistic assessment decision matrix is formed in Table S3.

Stage 2: Procedure to determine the criteria weights

No. of steps	Description of the procedure
Step 1.	**IVPF-WENSLO model for attaining the objective weight of each criterion**

Sub-step 1.1: Find the normalized decision matrix.
In this method, we first create the IVPF-score decision matrix and further normalize it as $N = (n_{ij})_{s\times t}$, where $n_{ij} = \frac{S(\tilde{\upsilon}_{ij})}{\sum_{i=1}^{s} S(\tilde{\upsilon}_{ij})}$ and $S(\tilde{\upsilon}_{ij})$ denotes the score value of an element '$\tilde{\upsilon}_{ij}$' of an IVPFADM, $i = 1, 2, \ldots, s$ and $j = 1, 2, \ldots, t$.
Sub-step 1.2: Specify the class interval of criterion.
With the use of Sturges' rule (Pamucar *et al.*, 2023), the class interval of each criterion is obtained via Eq. (27):

$$\Delta n_j = \frac{\max_{i=1,2,\ldots,s} n_{ij} - \min_{i=1,2,\ldots,s} n_{ij}}{1 + 3.332\log(s)} \quad (j = 1, 2, \ldots, t). \tag{27}$$

Sub-step 1.3: Estimate the criterion slope.
In line with Eq. (27), the slope of each criterion is obtained by means of Eq. (28):

$$\tan\zeta_j = \frac{\sum_{i=1}^{s} n_{ij}}{(s-1)\Delta n_j} \quad (j = 1, 2, \ldots, t). \tag{28}$$

Sub-step 1.4: Compute the criterion envelope.
Considering the class interval of the criterion, the envelope is achieved as the total Euclidean distance across consecutive normalized values and calculated through Eq. (29):

$$O_j = \sum_{i=1}^{s} \sqrt{(n_{(i+1)j} - n_{ij})^2 + (\Delta n_j)^2} \quad (j = 1, 2, \ldots, t). \tag{29}$$

Sub-step 1.5: Compute the ratio of envelope-slope.
Taking into account the criterion slope '$\tan\phi_j$' and criterion envelope 'O_j', the envelope-slope ratio of each criterion is determined with the help of Eq. (30):

$$\alpha_j = \frac{O_j}{\tan\zeta_j} \quad (j = 1, 2, \ldots, t). \tag{30}$$

Sub-step 1.6: Obtain the objective weight 'w_j^o' of jth criterion using Eq. (31):

$$w_j^o = \frac{\alpha_j}{\sum_{i=1}^{t} \alpha_j} \quad (j = 1, 2, \ldots, t). \tag{31}$$

Step 2.	**IVPF-RANCOM model for finding the subjective weight of each criterion.**

Sub-step 2.1: Create the aggregated column matrix.
The DMs offer their linguistic opinions regarding the performance of each criterion and further converted into IVPFNs. Consequently, construct the aggregated column matrix $C_A = (c_j)_{1\times t}$, where c_j is an aggregated IVPFN, calculated through IVPFWA operator.
Sub-step 2.2: Arrange the criteria positions.
Obtain the score value of each element of aggregated column matrix and then arrange the criteria positions accordingly.

Sub-step 2.3: Create the RANCOM matrix.
Considering the pairwise comparison of criteria preferences, create the RANCOM matrix $M = (\beta_{lj})_{t\times t}$, where

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ \vdots \\ v_t \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & \cdots & v_t \end{array} \\ \begin{bmatrix} \beta_{11} & \beta_{11} & \cdots & \beta_{1t} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{t1} & \beta_{t2} & \cdots & \beta_{tt} \end{bmatrix} \end{array} \quad \text{and} \quad \beta_{lj} = \begin{cases} 1, \text{ if } S(c_j) < S(c_l), \\ 0.5, \text{ if } S(c_j) = S(c_l), \\ 0, \text{ if } S(c_j) > S(c_l). \end{cases} \tag{32}$$

Sub-step 2.4: Find the summed weight of jth criterion using Eq. (33):

$$\sigma_j = \sum_{l=1}^{t} \beta_{jl} \quad (j = 1, 2, \ldots, t). \tag{33}$$

Sub-step 2.5: Obtain the subjective weight 'w_j^s' of jth criterion through Eq. (34):

$$w_j^s = \frac{\sigma_j}{\sum_{i=1}^{t} \sigma_j} \quad (j = 1, 2, \ldots, t). \tag{34}$$

Step 3. **Criteria weights through IVPF-WENSLO-RANCOM model.**

Combining the objective and subjective weights of criteria, derive the combined weight of each criterion using Eq. (35):

$$w_j = \varsigma w_j^O + (1 - \varsigma) w_j^S, \tag{35}$$

where $\varsigma \in [0, 1]$ stands for the decision-precision coefficient.

***Step* 2:** To find the experts' weights; initially, an average IVPFDM $\bar{F} = (\bar{\upsilon}_{ij}^{(k)})_{5\times 4}$ is constructed in Table 6, and then an entropy measure of each element of the average IVPFDM is computed and shown in Table 7. Further, the computed entropy value is normalized and presented in Table 7. Utilizing the normalized entropy values in Eq. (25), the weight of each expert is calculated and outlined in Table 7.

***Step* 3:** Using the DMs' weights in the IVPFWA operator, an IVPFADM is created via Eq. (26) and represented in Table 8, where each element denotes the collective IVPF-assessment rating of hospitals with respect to shortlisted evaluation criteria.

Stage 2: Procedure to determine the criteria weights

***Step* 1:** To determine the objective weight through the IVPF-WENSLO approach, we need to first find the IVPF decision matrix through Eq. (10). Further, the IVPF-score decision matrix is normalized, and the results are presented in Table 9.

Considering Sturges' rule (Pamucar *et al.*, 2023) given in Eq. (27), we determine the class interval (Δn_j) of each criterion and given as $\Delta n_1 = 0.016$, $\Delta n_2 = 0.016$, $\Delta n_3 = 0.025$, $\Delta n_4 = 0.017$, $\Delta n_5 = 0.013$, $\Delta n_6 = 0.019$, $\Delta n_7 = 0.018$, $\Delta n_8 = 0.013$, $\Delta n_9 =$

Stage 3: Procedure of an IVPF extension of CRADIS method

No. of steps **Description of the procedure**

Step 1. **Determine the IVPF-ideal solution and IVPF anti-ideal solution**

With the use of Eq. (36) and Eq. (37), the ideal and anti-ideal solutions are computed respectively, in terms of IVPFNs:

$$t\alpha_j^+ = \begin{cases} ([\max\limits_{i=1,2,\ldots,s} \mu_{ij}^-, \min\limits_{i=1,2,\ldots,s} \nu_{ij}^-], [\max\limits_{i=1,2,\ldots,s} \mu_{ij}^+, \min\limits_{i=1,2,\ldots,s} \nu_{ij}^+]), j \in v_b, \\ ([\min\limits_{i=1,2,\ldots,s} \mu_{ij}^-, \max\limits_{i=1,2,\ldots,s} \nu_{ij}^-], [\min\limits_{i=1,2,\ldots,s} \mu_{ij}^+, \max\limits_{i=1,2,\ldots,s} \nu_{ij}^+]), j \in v_n, \end{cases} \tag{36}$$

$$\alpha_j^- = \begin{cases} ([\min\limits_{i=1,2,\ldots,s} \mu_{ij}^-, \max\limits_{i=1,2,\ldots,s} \nu_{ij}^-], [\min\limits_{i=1,2,\ldots,s} \mu_{ij}^+, \max\limits_{i=1,2,\ldots,s} \nu_{ij}^+]), j \in v_b, \\ ([\max\limits_{i=1,2,\ldots,s} \mu_{ij}^-, \min\limits_{i=1,2,\ldots,s} \nu_{ij}^-], [\max\limits_{i=1,2,\ldots,s} \mu_{ij}^+, \min\limits_{i=1,2,\ldots,s} \nu_{ij}^+]), j \in v_n. \end{cases} \tag{37}$$

Step 2. **Compute the distances** (d_j^+) and (d_j^-)

In line with the proposed distance measure, calculate the deviation relation between each alternative and IVPF-ideal solution using Eq. (38):

$$d_j^+ = \sum_{j=1}^{t} w_j \sqrt{\left[\begin{array}{l} \left(\frac{(\mu_{ij}^-)^2+(\mu_{ij}^+)^2-(\nu_{ij}^-)^2-(\nu_{ij}^+)^2}{4} - \frac{(\mu_{j_{IS}}^-)^2+(\mu_{j_{IS}}^+)^2-(\nu_{j_{IS}}^-)^2-(\nu_{j_{IS}}^+)^2}{4}\right)^2 \\ +\frac{1}{3}\left(\frac{(\mu_{ij}^-)^2+(\mu_{ij}^-)^2+(\nu_{ij}^-)^2+(\nu_{ij}^-)^2}{4} - \frac{(\mu_{j_{IS}}^-)^2+(\mu_{j_{IS}}^-)^2+(\nu_{j_{IS}}^-)^2+(\nu_{j_{IS}}^-)^2}{4}\right)^2 \end{array}\right]}. \tag{38}$$

Similarly, compute the deviation relation between each alternative and IVPF anti-ideal solution through Eq. (39):

$$d_j^- = \sum_{j=1}^{t} w_j \sqrt{\left[\begin{array}{l} \left(\frac{(\mu_{ij}^-)^2+(\mu_{ij}^+)^2-(\nu_{ij}^-)^2-(\nu_{ij}^+)^2}{4} - \frac{(\mu_{j_{AIS}}^-)^2+(\mu_{j_{AIS}}^+)^2-(\nu_{j_{AIS}}^-)^2-(\nu_{j_{AIS}}^+)^2}{4}\right)^2 \\ +\frac{1}{3}\left(\frac{(\mu_{ij}^-)^2+(\mu_{ij}^-)^2+(\nu_{ij}^-)^2+(\nu_{ij}^-)^2}{4} - \frac{(\mu_{j_{AIS}}^-)^2+(\mu_{j_{AIS}}^-)^2+(\nu_{j_{AIS}}^-)^2+(\nu_{j_{AIS}}^-)^2}{4}\right)^2 \end{array}\right]}. \tag{39}$$

Step 3. **Find the utility functions**

Considering the deviation relations from Step 2, estimate the utility functions (U_i^+) and (U_i^-) from Eqs. (40) and (41):

$$U_i^+ = \min(d_i^+)/d_i^+ \quad (i = 1, 2, \ldots, s), \tag{40}$$

$$U_i^- = d_i^- / \max(d_i^-) \quad (i = 1, 2, \ldots, s). \tag{41}$$

Step 4. **Calculate the collective measure**

Taking into account the utility functions (U_i^+) and (U_i^-), find the collective measure of each alternative through Eq. (42), where '$\lambda \in [0, 1]$' stands for the strategy coefficient:

$$U_i = \lambda U_i^+ + (1 - \lambda)U_i^- \quad (i = 1, 2, \ldots, s). \tag{42}$$

Step 5. **Determine the alternatives' ranking**

On the basis of the determined collective measure of each alternative, estimate the ranking order of alternatives.

Table 4
Details of the criteria for assessing the quality of service of five hospitals in Patna.

Service quality dimensions	Criteria	References
Patient Management System	Hospital location (v_1)	(Rehman *et al.*, 2023; Alshouha *et al.*, 2024)
	Emergency parking (v_2)	(Aaronson *et al.*, 2015; Rehman *et al.*, 2023)
	Public transport (v_3)	(Yucesan and Gul, 2020; Rehman *et al.*, 2023)
	Ambulance service (v_4)	(Yucesan and Gul, 2020; Rehman *et al.*, 2023)
	Operating hour (v_5)	(Kondasani and Panda, 2015; Yucesan and Gul, 2020)
Patient relationship management	Appointment system (v_6)	(Kondasani and Panda, 2015; Rehman *et al.*, 2023)
	Registration process (v_7)	(Amin and Nasharuddin, 2013; Rehman *et al.*, 2023)
	Walk-in facility (v_8)	(Aliman and Mohamad, 2013; Rehman *et al.*, 2023)
	Prompt response (v_9)	(Rehman *et al.*, 2023)
	Effective communication (v_{10})	(Rehman *et al.*, 2023; Alshouha *et al.*, 2024)
	Managerial dependence (v_{11})	(Aliman and Mohamad, 2013; Rehman *et al.*, 2023)
	Reception assistance (v_{12})	(Rehman *et al.*, 2023)
Patient-physician relationship	Waiting time (v_{13})	(Aliman and Mohamad, 2013; Rehman *et al.*, 2023)
	On-demand doctors (v_{14})	(Aliman and Mohamad, 2013; Rehman *et al.*, 2023)
	Personalized care (v_{15})	(Yucesan and Gul, 2020; Rehman *et al.*, 2023)
	Medical examination (v_{16})	(Rehman *et al.*, 2023)
	Medication expertise (v_{17})	(Amin and Nasharuddin, 2013; Rehman *et al.*, 2023)
	Trustworthy care (v_{18})	(Aliman and Mohamad, 2013; Rehman *et al.*, 2023)

Table 5
Information about the members of the DEs' committee.

DM	Professional position	Experience	Primary perspective in the evaluation
g_1	Senior Medical Superintendent / Hospital Administrator	15–20 years	Hospital administration, management, patient-service delivery, and organizational operations
g_2	Senior Physician / Consultant	12–18 years	Clinical service delivery, physician-patient interaction, and clinical aspects of service quality
g_3	Hospital Quality Manager / NABH Quality Professional	10–15 years	Healthcare quality management, quality assurance, accreditation, and service quality monitoring
g_4	Public Health / Health Services Management Professor	12–18 years	Public health, health-services management, healthcare-system evaluation, and academic expertise

Table 6
Average IVPF decision matrix for finding the experts' weights.

Alternatives	g_1	g_2	g_3	g_4
A_1	([0.633, 0.751], [0.412, 0.523])	([0.579, 0.717], [0.472, 0.590])	([0.511, 0.625], [0.527, 0.629])	([0.472, 0.589], [0.565, 0.660])
A_2	([0.628, 0.754], [0.415, 0.534])	([0.606, 0.738], [0.447, 0.554])	([0.645, 0.774], [0.390, 0.530])	([0.500, 0.619], [0.535, 0.632])
A_3	([0.492, 0.622], [0.543, 0.652])	([0.545, 0.677], [0.487, 0.608])	([0.494, 0.603], [0.523, 0.647])	([0.486, 0.627], [0.559, 0.657])
A_4	([0.638, 0.762], [0.397, 0.523])	([0.534, 0.669], [0.506, 0.625])	([0.460, 0.583], [0.570, 0.674])	([0.495, 0.631], [0.548, 0.652])
A_5	([0.603, 0.722], [0.436, 0.560])	([0.571, 0.701], [0.480, 0.584])	([0.530, 0.652], [0.509, 0.611])	([0.521, 0.664], [0.522, 0.625])

Table 7
Weight of DEs based on the proposed IVPF-entropy.

$E(\bar{\upsilon}_{ik})$	g_1	g_2	g_3	g_4
A_1	0.799	0.892	0.992	0.931
A_2	0.804	0.843	0.772	0.98
A_3	0.966	0.944	0.969	0.956
A_4	0.785	0.967	0.915	0.969
A_5	0.854	0.906	0.972	0.98
$\bar{E}(\bar{\upsilon}_{ij})$	g_1	g_2	g_3	g_4
A_1	0.827	0.833	1	0.813
A_2	0.923	0.871	0.976	1
A_3	1	0.778	0.977	0.922
A_4	0.95	1	0.976	0.988
A_5	0.827	0.833	1	0.813
ω_k	0.229	0.253	0.250	0.267

0.009, $\Delta n_{10} = 0.015$, $\Delta n_{11} = 0.014$, $\Delta n_{12} = 0.015$, $\Delta n_{13} = 0.011$, $\Delta n_{14} = 0.009$, $\Delta n_{15} = 0.015$, $\Delta n_{16} = 0.006$, $\Delta n_{17} = 0.021$, and $\Delta n_{18} = 0.012$. In the next step, we compute the slope value ($\tan \zeta_j$) of each criterion using Eq. (28), and presented as $\tan \zeta_1 = 15.25$, $\tan \zeta_2 = 15.757$, $\tan \zeta_3 = 10.135$, $\tan \zeta_4 = 14.291$, $\tan \zeta_5 = 19.708$, $\tan \zeta_6 = 13.397$, $\tan \zeta_7 = 13.975$, $\tan \zeta_8 = 19.358$, $\tan \zeta_9 = 27.703$, $\tan \zeta_{10} = 16.928$, $\tan \zeta_{11} = 18.315$, $\tan \zeta_{12} = 16.223$, $\tan \zeta_{13} = 23.747$, $\tan \zeta_{14} = 28.823$, $\tan \zeta_{15} = 17.019$, $\tan \zeta_{16} = 39.079$, $\tan \zeta_{17} = 12.031$, and $\tan \zeta_{18} = 20.67$.

Based on Eq. (29), the envelope value (O_j) of each criterion is obtained as $O_1 = 0.151$, $O_2 = 0.161$, $O_3 = 0.139$, $O_4 = 0.123$, $O_5 = 0.119$, $O_6 = 0.104$, $O_7 = 0.16$, $O_8 = 0.092$, $O_9 = 0.057$, $O_{10} = 0.112$, $O_{11} = 0.102$, $O_{12} = 0.078$, $O_{13} = 0.048$, $O_{14} = 0.052$, $O_{15} = 0.136$, $O_{16} = 0.039$, $O_{17} = 0.08$ and $O_{18} = 0.077$. Further, the ratio of envelope-slope for each criterion is calculated via Eq. (30) and given as $\alpha_1 = 0.0099$, $\alpha_2 = 0.0102$, $\alpha_3 = 0.0137$, $\alpha_4 = 0.0086$, $\alpha_5 = 0.0061$, $\alpha_6 = 0.0077$, $\alpha_7 = 0.0115$, $\alpha_8 = 0.0047$, $\alpha_9 = 0.002$, $\alpha_{10} = 0.0066$, $\alpha_{11} = 0.0055$, $\alpha_{12} = 0.0048$, $\alpha_{13} = 0.002$, $\alpha_{14} = 0.0018$, $\alpha_{15} = 0.008$, $\alpha_{16} = 0.001$, $\alpha_{17} = 0.0067$ and $\alpha_{18} = 0.0037$. Finally, we derive the objective weight of each criterion with the help of Eq. (31), and thus, we get $w_1^o = 0.0862$, $w_2^o = 0.0892$, $w_3^o = 0.1195$, $w_4^o = 0.0749$, $w_5^o = 0.0528$, $w_6^o = 0.0676$, $w_7^o = 0.1$, $w_8^o = 0.0413$, $w_9^o = 0.0179$, $w_{10}^o = 0.0578$, $w_{11}^o = 0.0484$, $w_{12}^o = 0.0417$, $w_{13}^o = 0.0177$, $w_{14}^o = 0.0159$, $w_{15}^o = 0.0698$, $w_{16}^o = 0.0087$, $w_{17}^o = 0.0582$, and $w_{18}^o = 0.0325$.

***Step* 2:** To derive the subjective weight through the IVPF-RANCOM model, we need to collect the DMs' linguistic opinions regarding the performance of each criterion. Consequently, an aggregated column matrix is constructed through the IVPFWA operator (Eq. (4)), and their score values are further determined, given in Table 10. In accordance with the score values of the criteria, the rank of each criterion is attained. The last two columns of Table 10 present the score value and rank of each criterion, respectively.

Taking into account the pairwise comparison of criteria preferences, the RANCOM matrix is created with the utilization of Eq. (32); consequently, the summed weight of each

Table 8
Aggregated decision matrix for assessing the quality of service of five hospitals.

Criteria	Alternatives				
	A_1	A_2	A_3	A_4	A_5
v_1	([0.454, 0.585], [0.547, 0.648])	([0.617, 0.769], [0.425, 0.525])	([0.433, 0.564], [0.573, 0.673])	([0.501, 0.638], [0.522, 0.623])	([0.573, 0.722], [0.464, 0.565])
v_2	([0.667, 0.773], [0.374, 0.468])	([0.730, 0.836], [0.303, 0.391])	([0.529, 0.675], [0.493, 0.594])	([0.713, 0.837], [0.304, 0.441])	([0.509, 0.648], [0.516, 0.617])
v_3	([0.649, 0.785], [0.390, 0.518])	([0.741, 0.852], [0.268, 0.421])	([0.511, 0.650], [0.514, 0.615])	([0.489, 0.629], [0.541, 0.643])	([0.447, 0.580], [0.599, 0.692])
v_4	([0.661, 0.795], [0.371, 0.497])	([0.628, 0.761], [0.397, 0.524])	([0.590, 0.723], [0.424, 0.551])	([0.494, 0.635], [0.537, 0.639])	([0.730, 0.841], [0.299, 0.387])
v_5	([0.527, 0.673], [0.495, 0.595])	([0.701, 0.823], [0.320, 0.461])	([0.661, 0.795], [0.371, 0.497])	([0.540, 0.670], [0.496, 0.622])	([0.656, 0.791], [0.378, 0.502])
v_6	([0.797, 0.889], [0.209, 0.319])	([0.705, 0.826], [0.315, 0.457])	([0.599, 0.732], [0.415, 0.543])	([0.525, 0.671], [0.497, 0.597])	([0.706, 0.817], [0.318, 0.407])
v_7	([0.489, 0.629], [0.541, 0.643])	([0.723, 0.837], [0.308, 0.397])	([0.581, 0.706], [0.437, 0.568])	([0.688, 0.798], [0.342, 0.433])	([0.556, 0.699], [0.486, 0.587])
v_8	([0.701, 0.823], [0.320, 0.461])	([0.756, 0.855], [0.264, 0.369])	([0.568, 0.715], [0.469, 0.570])	([0.635, 0.768], [0.391, 0.518])	([0.665, 0.783], [0.353, 0.501])
v_9	([0.236, 0.321], [0.778, 0.844])	([0.276, 0.371], [0.732, 0.810])	([0.299, 0.269], [0.691, 0.831])	([0.239, 0.335], [0.762, 0.838])	([0.210, 0.307], [0.788, 0.850])
v_{10}	([0.211, 0.307], [0.788, 0.850])	([0.222, 0.315], [0.774, 0.851])	([0.313, 0.412], [0.693, 0.783])	([0.245, 0.337], [0.763, 0.830])	([0.258, 0.357], [0.746, 0.824])
v_{11}	([0.274, 0.369], [0.734, 0.823])	([0.199, 0.293], [0.795, 0.860])	([0.243, 0.340], [0.758, 0.835])	([0.188, 0.274], [0.811, 0.875])	([0.191, 0.285], [0.801, 0.864])
v_{12}	([0.215, 0.312], [0.784, 0.849])	([0.197, 0.291], [0.796, 0.861])	([0.215, 0.312], [0.784, 0.849])	([0.296, 0.392], [0.709, 0.799])	([0.306, 0.405], [0.700, 0.789])
v_{13}	([0.215, 0.312], [0.784, 0.848])	([0.257, 0.356], [0.747, 0.824])	([0.296, 0.392], [0.709, 0.799])	([0.290, 0.388], [0.720, 0.797])	([0.298, 0.394], [0.708, 0.797])
v_{14}	([0.243, 0.340], [0.758, 0.835])	([0.293, 0.391], [0.716, 0.795])	([0.292, 0.390], [0.718, 0.796])	([0.310, 0.408], [0.697, 0.786])	([0.278, 0.374], [0.730, 0.808])
v_{15}	([0.523, 0.668], [0.497, 0.598])	([0.638, 0.777], [0.383, 0.506])	([0.527, 0.673], [0.495, 0.595])	([0.712, 0.837], [0.304, 0.442])	([0.590, 0.722], [0.424, 0.551])
v_{16}	([0.676, 0.781], [0.359, 0.451])	([0.675, 0.811], [0.354, 0.477])	([0.622, 0.743], [0.389, 0.516])	([0.584, 0.737], [0.448, 0.548])	([0.639, 0.772], [0.386, 0.514])
v_{17}	([0.412, 0.531], [0.592, 0.693])	([0.489, 0.628], [0.541, 0.643])	([0.626, 0.761], [0.424, 0.551])	([0.668, 0.792], [0.357, 0.501])	([0.688, 0.798], [0.342, 0.433])
v_{18}	([0.761, 0.863], [0.256, 0.358])	([0.711, 0.836], [0.305, 0.442])	([0.585, 0.711], [0.433, 0.564])	([0.665, 0.783], [0.353, 0.501])	([0.735, 0.857], [0.287, 0.421])

criterion is found through Eq. (33). Table 11 outlines the computational steps required to construct the RANCOM matrix and to calculate the summed criteria weights. In addition, the subjective weights of the criteria are obtained via Eq. (34) as $w_1^s = 0.071$, $w_2^s = 0.0525$, $w_3^s = 0.0833$, $w_4^s = 0.0957$, $w_5^s = 0.0772$, $w_6^s = 0.0401$, $w_7^s = 0.0463$, $w_8^s = 0.034$, $w_9^s = 0.0093$, $w_{10}^s = 0.0154$, $w_{11}^s = 0.0031$, $w_{12}^s = 0.0278$, $w_{13}^s = 0.0216$, $w_{14}^s = 0.0648$, $w_{15}^s = 0.0586$, $w_{16}^s = 0.108$, $w_{17}^s = 0.1019$, and $w_{18}^s = 0.0895$.

***Step* 3:** Applying Eq. (35), the combined weight of each criterion is calculated as the product of the "sum of ς" and "objective weight", as well as the product of the "complement of ς; i.e. $(1 - \varsigma)$" and "subjective weight of the criterion". For $\varsigma = 0.5$, the combined

Table 9
IVPF-score and normalized IVPF decision matrix for assessing the quality of service of five hospitals.

Criteria	Alternatives									
	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5
v_1	0.316	0.493	0.295	0.358	0.438	0.166	0.259	0.155	0.188	0.231
v_2	0.533	0.621	0.392	0.606	0.366	0.212	0.247	0.156	0.241	0.146
v_3	0.522	0.637	0.369	0.345	0.297	0.241	0.294	0.17	0.159	0.137
v_4	0.539	0.497	0.453	0.35	0.624	0.219	0.202	0.184	0.142	0.253
v_5	0.39	0.586	0.539	0.39	0.532	0.16	0.241	0.221	0.16	0.218
v_6	0.714	0.591	0.464	0.387	0.592	0.26	0.215	0.169	0.141	0.215
v_7	0.345	0.615	0.438	0.565	0.415	0.145	0.259	0.184	0.238	0.174
v_8	0.586	0.655	0.432	0.505	0.535	0.216	0.241	0.159	0.186	0.197
v_9	0.128	0.158	0.15	0.135	0.119	0.185	0.23	0.217	0.196	0.172
v_{10}	0.119	0.124	0.186	0.137	0.147	0.167	0.174	0.261	0.192	0.206
v_{11}	0.155	0.112	0.138	0.102	0.108	0.252	0.182	0.224	0.166	0.176
v_{12}	0.121	0.111	0.121	0.172	0.18	0.172	0.157	0.172	0.244	0.255
v_{13}	0.121	0.147	0.172	0.169	0.174	0.155	0.188	0.22	0.215	0.222
v_{14}	0.138	0.171	0.17	0.183	0.16	0.168	0.208	0.207	0.223	0.195
v_{15}	0.386	0.514	0.39	0.605	0.453	0.164	0.219	0.166	0.258	0.193
v_{16}	0.545	0.559	0.486	0.455	0.511	0.213	0.219	0.19	0.178	0.2
v_{17}	0.272	0.345	0.489	0.542	0.565	0.123	0.156	0.221	0.245	0.255
v_{18}	0.665	0.604	0.443	0.535	0.636	0.231	0.21	0.154	0.186	0.22

Table 10
Significance ratings of each criterion during the assessment of the quality of service of five hospitals.

Criteria	g_1	g_2	g_3	g_4	Aggregated IVPFNs	Score values	Rank
v_1	M	VH	MH	M	([0.584, 0.710], [0.434, 0.565])	0.441	7
v_2	ML	M	MH	H	([0.498, 0.638], [0.533, 0.635])	0.354	10
v_3	VH	MH	H	M	([0.623, 0.757], [0.402, 0.528])	0.492	5
v_4	M	H	MH	VH	([0.639, 0.772], [0.386, 0.514])	0.511	3
v_5	VH	ML	H	MH	([0.614, 0.750], [0.417, 0.545])	0.48	6
v_6	M	L	MH	M	([0.394, 0.512], [0.617, 0.708])	0.255	12
v_7	ML	M	H	L	([0.439, 0.570], [0.607, 0.699])	0.289	11
v_8	MH	M	L	ML	([0.368, 0.486], [0.644, 0.735])	0.232	13
v_9	ML	M	ML	ML	([0.329, 0.429], [0.673, 0.773])	0.198	17
v_{10}	M	L	ML	M	([0.337, 0.436], [0.671, 0.760])	0.204	16
v_{11}	L	VL	MH	ML	([0.321, 0.441], [0.697, 0.778])	0.194	18
v_{12}	VL	M	MH	L	([0.348, 0.466], [0.671, 0.752])	0.215	14
v_{13}	MH	ML	L	ML	([0.343, 0.462], [0.670, 0.760])	0.212	15
v_{14}	H	H	ML	MH	([0.553, 0.704], [0.488, 0.590])	0.415	8
v_{15}	MH	H	M	M	([0.506, 0.643], [0.519, 0.620])	0.363	9
v_{16}	H	H	M	VH	([0.664, 0.797], [0.368, 0.494])	0.542	1
v_{17}	VH	H	M	H	([0.656, 0.792], [0.378, 0.501])	0.533	2
v_{18}	H	M	MH	VH	([0.635, 0.767], [0.389, 0.518])	0.505	4

weight of the criterion is achieved via Eq. (35) and shown as $w_1 = 0.0786$, $w_2 = 0.0708$, $w_3 = 0.1014$, $w_4 = 0.0853$, $w_5 = 0.065$, $w_6 = 0.0538$, $w_7 = 0.0732$, $w_8 = 0.0377$, $w_9 = 0.0136$, $w_{10} = 0.0366$, $w_{11} = 0.0257$, $w_{12} = 0.0348$, $w_{13} = 0.0196$, $w_{14} = 0.0403$, $w_{15} = 0.0642$, $w_{16} = 0.0583$, $w_{17} = 0.0801$ and $w_{18} = 0.061$. Figure 2 presents the graphical representation of the criteria weighting values.

Table 11
Results of the IVPF-RANCOM model for assessing hospital service quality.

Criteria	RANCOM matrix																		σ_j
	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}	v_{16}	v_{17}	v_{18}	
v_1	0.5	1	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	11.5
v_2	0	0.5	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	8.5
v_3	1	1	0.5	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	13.5
v_4	1	1	1	0.5	1	1	1	1	1	1	1	1	1	1	1	0	0	1	15.5
v_5	1	1	0	0	0.5	1	1	1	1	1	1	1	1	1	1	0	0	0	12.5
v_6	0	0	0	0	0	0.5	0	1	1	1	1	1	1	0	0	0	0	0	6.5
v_7	0	0	0	0	0	1	0.5	1	1	1	1	1	1	0	0	0	0	0	7.5
v_8	0	0	0	0	0	0	0	0.5	1	1	1	1	1	0	0	0	0	0	5.5
v_9	0	0	0	0	0	0	0	0	0.5	0	1	0	0	0	0	0	0	0	1.5
v_{10}	0	0	0	0	0	0	0	0	1	0.5	1	0	0	0	0	0	0	0	2.5
v_{11}	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0	0	0	0	0	0.5
v_{12}	0	0	0	0	0	0	0	0	1	1	1	0.5	1	0	0	0	0	0	4.5
v_{13}	0	0	0	0	0	0	0	0	1	1	1	0	0.5	0	0	0	0	0	3.5
v_{14}	0	1	0	0	0	1	1	1	1	1	1	1	1	0.5	1	0	0	0	10.5
v_{15}	0	1	0	0	0	1	1	1	1	1	1	1	1	0	0.5	0	0	0	9.5
v_{16}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.5	1	1	17.5
v_{17}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0.5	1	16.5
v_{18}	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0.5	14.5

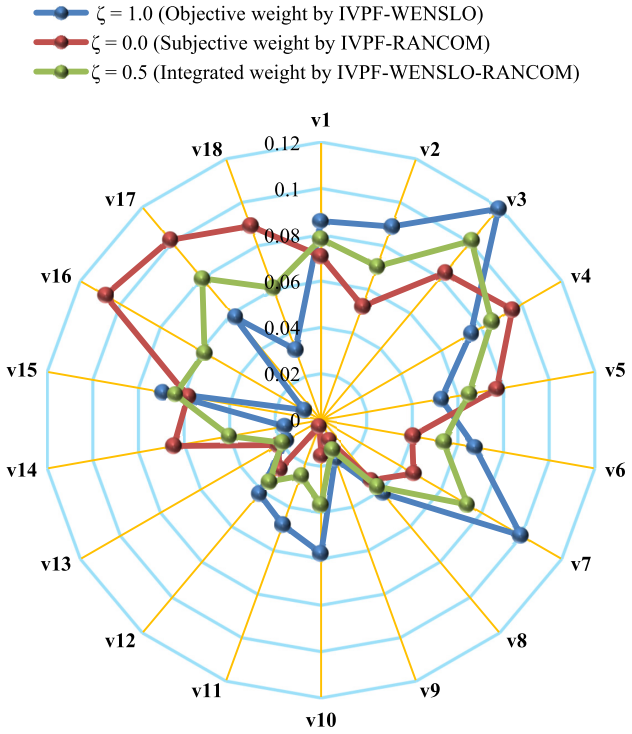


Fig. 2. Representation of criteria weight values.

Stage 3: Procedure for an IVPF extension of the CRADIS

***Step* 1:** Considering Table 7, we determine the IVPF-ideal solution using Eq. (36) and given as $\alpha_j^+ = \{([0.617, 0.769], [0.425, 0.525]), ([0.730, 0.836], [0.303, 0.391]), ([0.741, 0.852], [0.268, 0.421]), ([0.730, 0.841], [0.299, 0.387]), ([0.701, 0.823], [0.320, 0.461]), ([0.797, 0.889], [0.209, 0.319]), ([0.723, 0.837], [0.308, 0.397]), ([0.756, 0.855], [0.264, 0.369]), ([0.276, 0.371], [0.732, 0.810]), ([0.313, 0.412], [0.693, 0.783]), ([0.274, 0.369], [0.734, 0.823]), ([0.306, 0.405], [0.7, 0.789]), ([0.298, 0.394], [0.708, 0.797]), ([0.31, 0.408], [0.697, 0.786]), ([0.712, 0.837], [0.304, 0.442]), ([0.675, 0.811], [0.354, 0.477]), ([0.688, 0.798], [0.342, 0.433]), ([0.761, 0.863], [0.256, 0.358])\}$, where $j = 1, 2, \ldots, 18$.

Next, we calculate the IVPF anti-ideal solution through Eq. (37) and given as $\alpha_j^- = \{([0.433, 0.564], [0.573, 0.673]), ([0.509, 0.648], [0.516, 0.617]), ([0.447, 0.580], [0.599, 0.692]), ([0.494, 0.635], [0.537, 0.639]), ([0.527, 0.673], [0.495, 0.595]), ([0.525, 0.671], [0.497, 0.597]), ([0.489, 0.629], [0.541, 0.643]), ([0.568, 0.715], [0.469, 0.57]), ([0.21, 0.307], [0.788, 0.85]), ([0.211, 0.307], [0.788, 0.85]), ([0.188, 0.274], [0.811, 0.875]), ([0.197, 0.291], [0.796, 0.861]), ([0.215, 0.312], [0.784, 0.848]), ([0.243, 0.34], [0.758, 0.835]), ([0.523, 0.668], [0.497, 0.598]), ([0.584, 0.737], [0.448, 0.548]), ([0.412, 0.531], [0.592, 0.693]), ([0.585, 0.711], [0.433, 0.564])\}$, where $j = 1, 2, \ldots, 18$.

***Step* 2:** With the utilization of the proposed distance measure, the deviation relation between each alternative and the IVPF-ideal solution is attained through Eq. (38), and given as $d_1^+ = 0.1174$, $d_2^+ = 0.0506$, $d_3^+ = 0.143$, $d_4^+ = 0.1185$, and $d_5^+ = 0.1005$. Similarly, the deviation relation between each alternative and the IVPF anti-ideal solution is computed via Eq. (39) and presented as $d_1^- = 0.0937$, $d_2^- = 0.1604$, $d_3^- = 0.0697$, $d_4^- = 0.0946$, and $d_5^- = 0.1119$.

***Step* 3:** Considering the deviation relations of above step, we compute the utility functions U_i^+ and U_i^- for each alternative A_i using Eq. (41) and Eq. (42), where $i = 1, 2, \ldots, 5$, and presented as $U_1^+ = 0.431$, $U_2^+ = 1.0$, $U_3^+ = 0.354$, $U_4^+ = 0.427$, $U_5^+ = 0.504$, $U_1^- = 0.584$, $U_2^- = 1.0$, $U_3^- = 0.434$, $U_4^- = 0.59$, and $U_5^- = 0.698$.

***Step* 4:** Using Eq. (42), we obtain the collective measure (for strategy coefficient $\lambda = 0.5$) for each alternative A_i, where $i = 1, 2, \ldots, 5$, $U_1 = 0.5077$, $U_2 = 1.0$, $U_3 = 0.3942$, $U_4 = 0.5085$, and $U_5 = 0.6007$.

***Step* 5:** The collective measure of each option allows for deriving the prioritization among options, which is $A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$. Thus, the option "Hospital-2 (A_2)" has the maximum preference in terms of better service quality.

6. Results Validation

In this section, we conduct sensitivity and comparative tests to confirm the stability and robustness of the outcomes.

Table 12
Results of the sensitivity analysis over varying values of the coefficient (ς).

	A_1	A_2	A_3	A_4	A_5	Ranking order
$\varsigma = 0.0$ (IVPF-RANCOM)	0.5343	1	0.427	0.5231	0.7188	$A_2 \succ A_5 \succ A_1 \succ A_4 \succ A_3$
$\varsigma = 0.1$	0.5288	1	0.4202	0.5201	0.6928	$A_2 \succ A_5 \succ A_1 \succ A_4 \succ A_3$
$\varsigma = 0.2$	0.5234	1	0.4135	0.5172	0.668	$A_2 \succ A_5 \succ A_1 \succ A_4 \succ A_3$
$\varsigma = 0.3$	0.5181	1	0.4069	0.5143	0.6445	$A_2 \succ A_5 \succ A_1 \succ A_4 \succ A_3$
$\varsigma = 0.4$	0.5129	1	0.4005	0.5113	0.6221	$A_2 \succ A_5 \succ A_1 \succ A_4 \succ A_3$
$\varsigma = 0.5$ (IVPF-WENSLO-RANCOM)	0.5077	1	0.3942	0.5085	0.6007	$A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$
$\varsigma = 0.6$	0.5027	1	0.388	0.5056	0.5802	$A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$
$\varsigma = 0.7$	0.4977	1	0.382	0.5028	0.5605	$A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$
$\varsigma = 0.8$	0.4928	1	0.376	0.4999	0.5417	$A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$
$\varsigma = 0.9$	0.488	1	0.3702	0.4971	0.5236	$A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$
$\varsigma = 1.0$ (IVPF-WENSLO)	0.4833	1	0.3645	0.4944	0.5061	$A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$

6.1. *Sensitivity Analysis*

This subsection considers the following two cases to examine the influence of fluctuations in the criteria weighting parameter (ς) and strategy coefficient (λ) on the final ranking output of the IVPF-WENSLO-RANCOM-CRADIS framework. The first case underlines the rationale for carrying out sensitivity analysis on the criteria weights when ranking the hospitals. The second case spotlights the rationale for carrying out sensitivity analysis with respect to the strategy coefficient when determining the hospital alternatives' ranking.

***Case I* (*Analysing the influence of varying values of the criteria weighting parameter* ς):** In this case, we investigate the degrees of collective measure over the varying values of the criteria weighting parameter (ς). To this aim, we consider the different values of the criteria weighting parameter (ς), ranging from $\varsigma = 0$ to $\varsigma = 1$. In accordance with the diverse values of ς, the collective measure of each alternative hospital is determined and shown in Table 12. Moreover, Fig. 3 demonstrates the results of the sensitivity investigation over the criteria weighting parameter (ς). From Table 12 and Fig. 3, we can observe that the ranking result varies for $\varsigma = 0$ to $\varsigma = 0.4$, while there is no change in the ranking of these hospitals for $\varsigma = 0.5$ to $\varsigma = 1$. Thus, the option "Hospital-2 (A_2)" has the maximum preference for different values of ς.

***Case II* (*Examining the impact of the strategy coefficient* λ):** In this case, we observe the effect of the possible variations in λ values on the assessment of service quality of hospitals. To this end, we consider various values of the strategy parameter (λ) between $\lambda = 0$ to $\lambda = 1$, with a step size of '0.1'. Table 13 and Fig. 4 present different scenarios taking into account $\lambda = 0.0$ to $\lambda = 1.0$. From Table 13 and Fig. 4, we can find that there is no change in the ranking of these hospitals for $\lambda = 0$ to $\lambda = 0.5$, while the ranking result varies for $\lambda = 0.6$ to $\lambda = 1$. However, the option "Hospital-2 (A_2)" has the maximum preference for different values of the strategy parameter (λ).

Based on these two cases of sensitivity analysis, we observe that the potential scores of the alternatives' collective measures display variations, but it should be noted that the "Hospital-2 (A_2)" has the maximum preference among the others in terms of service quality assessment of hospitals across both scenarios. This proves that the proposed MCGDM methodology has good robustness.

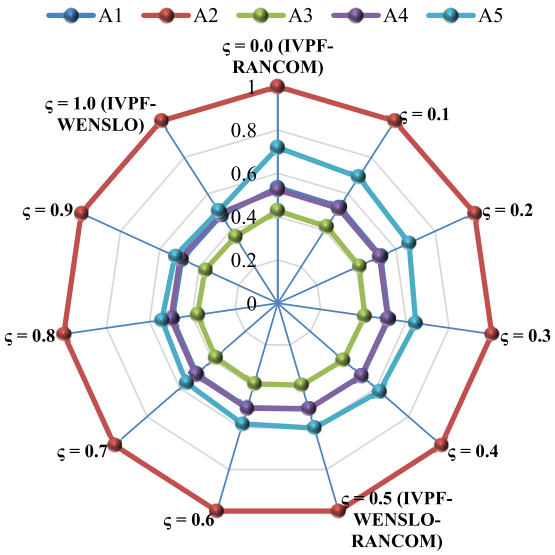


Fig. 3. Sensitivity results with respect to the criteria weighting parameter.

Table 13
Performance degree of options over strategy parameter (λ).

	A_1	A_2	A_3	A_4	A_5	Ranking order
$\lambda = 0.0$ (Utility measure by IVPFAIS)	0.584	1.0	0.434	0.59	0.698	$A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$
$\lambda = 0.1$	0.5687	1.0	0.426	0.5737	0.6786	$A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$
$\lambda = 0.2$	0.5534	1.0	0.418	0.5574	0.6592	$A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$
$\lambda = 0.3$	0.5381	1.0	0.41	0.5411	0.6398	$A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$
$\lambda = 0.4$	0.5228	1.0	0.402	0.5248	0.6204	$A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$
$\lambda = 0.5$ (Collective measure)	0.5075	1.0	0.394	0.5085	0.601	$A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$
$\lambda = 0.6$	0.4922	1.0	0.386	0.4922	0.5816	$A_2 \succ A_5 \succ A_4 \approx A_1 \succ A_3$
$\lambda = 0.7$	0.4769	1.0	0.378	0.4759	0.5622	$A_2 \succ A_5 \succ A_1 \succ A_4 \succ A_3$
$\lambda = 0.8$	0.4616	1.0	0.37	0.4596	0.5428	$A_2 \succ A_5 \succ A_1 \succ A_4 \succ A_3$
$\lambda = 0.9$	0.4463	1.0	0.362	0.4433	0.5234	$A_2 \succ A_5 \succ A_1 \succ A_4 \succ A_3$
$\lambda = 1.0$ (Utility measure by IVPFIS)	0.431	1.0	0.354	0.427	0.504	$A_2 \succ A_5 \succ A_1 \succ A_4 \succ A_3$

6.2. *Practical Implications of the Study Based on Variation of Ranking*

The sensitivity assessment reveals that the developed IVPF-based MCDM framework offers stable outcomes over diverse parameter settings such as criteria weighting parameter (ς) and strategy coefficient (λ). However, the experimental changes in the relative places of some hospitals over specific parameter arrangements also offer valuable real-world material for healthcare managers. A change in preference does not inevitably specify a weakness of the developed approach; rather, it reflects the diverse priorities allocated to the assessment criteria through the criteria weighting parameter (ς) and strategy coefficient (λ).

From a practical viewpoint, hospitals that sustain a consistently higher prioritization across the majority of parameter configurations can be viewed as indicating relatively sta-

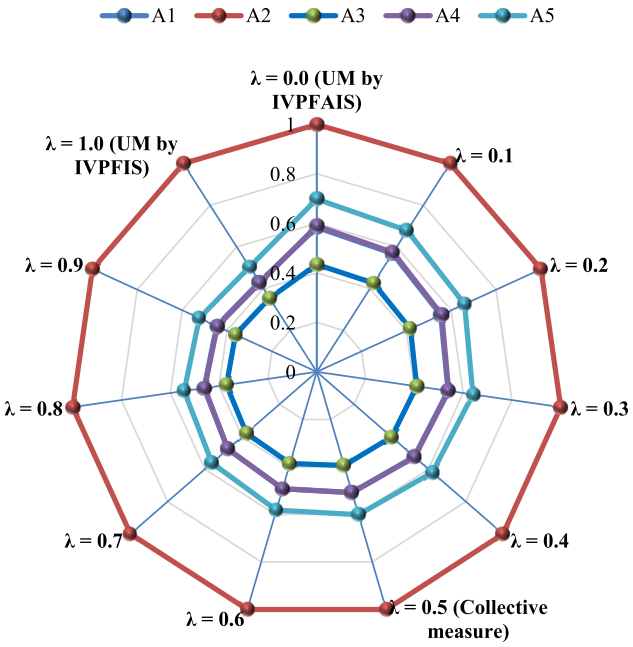


Fig. 4. Sensitivity results with respect to strategy parameter (λ).

ble performance across diverse interpretations of service quality significance. Such stability indicates that their overall performance is not strongly reliant on a specific parameter condition. On the other hand, hospitals whose positions change noticeably with the criteria weighting parameter (ς) and strategy coefficient (λ) should be interpreted with greater consideration because their relative outcome is more sensitive to changes in the importance or treatment of the assessment criteria in terms of better service quality.

This result is mainly appropriate for hospital administrators and healthcare policymakers. If any hospital changes from a higher to a lower preference when the significance rating of a specific service quality criterion is increased, this may indicate that the hospital has comparatively weaker performance on that criterion. Such an outcome can support administrators in recognizing areas needing targeted improvement rather than relying solely on the overall prioritization. For instance, variation in ranking associated with patient-management, patient-relationship, or patient-physician relationship criteria can be utilized to recognize which features of service quality have a greater influence on the hospital's overall position.

Thus, the sensitivity assessment can assist not only as a validation mechanism for the robustness of the developed approach but also as a managerial analytical procedure. Stable rankings offer confidence in the steadiness of the decision results, while changes in preference orders highlight hospitals and service quality dimensions that permit further investigation. In real-world applications, hospital DEs can utilize these outcomes to discriminate between structural performance differences and priority-dependent performance differences, by this means supporting more focused quality-improvement interventions.

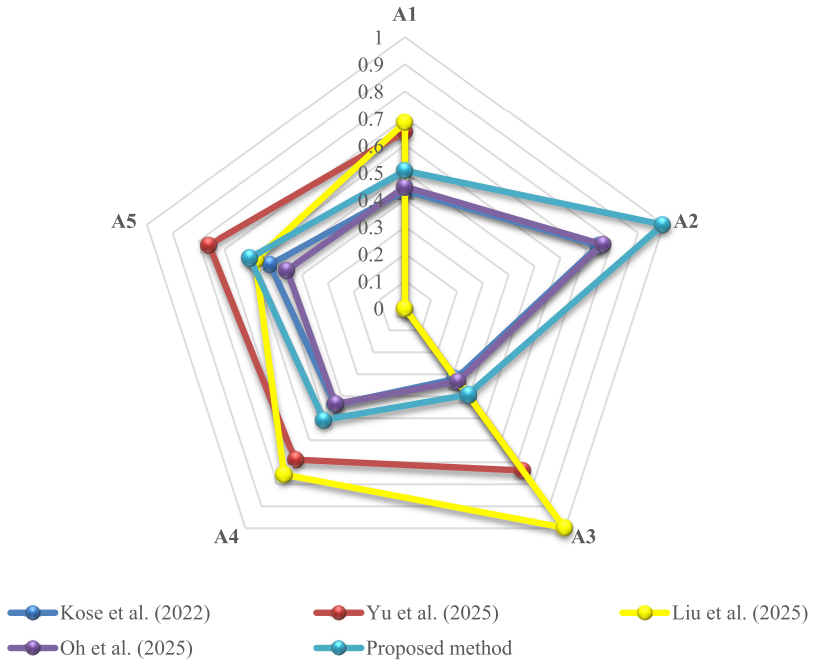


Fig. 5. Ranking results of hospitals by different IVPF methods.

6.3. *Comparative Analysis*

Here, a comparative analysis is performed to assess the robustness of the developed methodology over the existing IVPF information-based MCDM methods. To this end, we selected the previously developed methods from distance and score function-based perspectives and applied them to the aforesaid case study to find a suitable hospital in terms of 18 service criteria. For this purpose, some of the well-known PF information-based decision-making methods, including an extension of the TOPSIS method under the IVPF environment (Kose *et al.*, 2022), score function-based IVPF-TOPSIS (Oh *et al.*, 2025), IVPF-VIKOR (Yu *et al.*, 2025), IVPF distance-based extended inferior ratio method (Liu *et al.*, 2025), are selected and applied to the aforesaid case study. Details of the comparative analysis are given in Supplementary Material. Finally, Fig. 5 demonstrates the ranking orders of hospitals attained by the suggested and previous models.

The main advantages of the proposed IVPF-WENSLO-RANCOM-CRADIS model are presented as follows:

i. This method uses a novel entropy to obtain the significance values of DMs, as the opinions of DMs are very important in the real-life hospital service quality assessment problem, while existing studies related to healthcare service quality evaluation ignore the DMs' weights.
ii. Using the WENSLO and RANCOM models, the proposed method combines the objective and subjective weights of criteria during the evaluation of hospital service quality.

This means that the present method not only considers the quantitative data for determining the criterion's objective weight through the IVPF-WENSLO model but also involves the experts' preferences during the computation of the subjective weight of the criterion via the IVPF-RANCOM model.

iii. The proposed score function used in the IVPF-WENSLO model overcomes the limitations of existing score functions (Peng and Yang, 2016; Garg, 2016; 2017; Peng and Li, 2019; Alrasheedi and Jeevaraj, 2023; Rani *et al.*, 2023); thus, the ranking result can be more effective than existing studies.

iv. The proposed method combines the DMs and criteria weighting tools with the IVPF extension of the CRADIS model, in which the deviation relation from the ideal solutions to the alternatives is computed through a novel distance measure. Thus, the results obtained by the developed IVPF-WENSLO-RANCOM-CRADIS model are more effective and robust than existing methods (Kose *et al.*, 2022; Oh *et al.*, 2025; Yu *et al.*, 2025; Liu *et al.*, 2025).

7. Conclusions and Recommendations

Given the increasing importance of evaluating healthcare service quality, this work proposed a hybrid decision support framework within the context of IVPFSs, as such types of real-life problems contain uncertain and imprecise information. Before developing the algorithm, several formulas were introduced, including entropy measures, score functions, and distance measures. These formulae demonstrated significant advantages in overcoming the limitations of existing entropy, score function, and distance measures within the IVPF environment. Furthermore, we explored the application of entropy, score function, and distance measure in the development of the IVPF-WENSLO-RANCOM-CRADIS model. The DMs' weights were calculated through an entropy-based model, while the criteria weights were determined by an integrated WENSLO-RANCOM model. The combined criteria weights identify public transport, ambulance service, and medication expertise as the leading priorities in the evaluated setting, suggesting that hospital managers and policymakers should consider accessibility, emergency transport, and medication-related expertise when planning service improvements and allocating resources. Based on these steps, the alternatives' ranking was determined via an IVPF extension of the CRADIS model. Moreover, this methodology was implemented in a case study of healthcare service quality evaluation of five hospitals in Patna, the most populous city of Bihar (India), which proves its applicability and effectiveness. The insights gained from this work can support policymakers in designing service quality policies that rank healthcare services based on performance. Furthermore, sensitivity and comparative analyses were executed to demonstrate the stability and robustness of the developed methodology, respectively. The method simultaneously ranked hospitals according to 18 service criteria and calculated the combined objective-subjective weights of the criteria through the IVPF-WENSLO-RANCOM model together with DMs' weighting via the IVPF-entropy-based model. Moreover, there were obvious advantages of the suggested methodology in comparison with existing fuzzy MCDM methods related to healthcare service quality evaluation problems.

Nevertheless, this study has some limitations. First, there is no discussion about the interrelationships of criteria. Second, the considered criteria and DMs are very limited in this study. Third, it is necessary to separately appraise the quality of service of private and public hospitals. In future studies, we aim to overcome the limitations identified in this work. Specifically, machine learning and optimization methods may be combined with fuzzy information to render a more comprehensive assessment of hospitals' overall service quality.

Supplementary Material

Tables S1–S3 and detailed comparative analysis are included in Supplementary file.

Funding

The authors express their gratitude to the Deanship of Research and Graduate Studies at King Khalid University for supporting this research via the Large Group Project under grant number RGP2/587/46.

References

Aaronson, E.L., Marsh, R.H., Guha, M., Schuur, J.D., Rouhani, S.A. (2015). Emergency department quality and safety indicators in resource-limited settings: an environmental survey. *International Journal of Emergency Medicine*, 8, 39.

Akdag, H., Kalaycı, T., Karagöz, S., Zülfikar, H., Giz, D. (2014). The evaluation of hospital service quality by fuzzy MCDM. *Applied Soft Computing*, 23, 239–248.

Alam, M.F.B., Bari, A.B.M.M., Tushar, S.R., Kabir, K.M.A. (2025). An interval-valued Pythagorean fuzzy approach to mitigate traffic congestion in densely populated cities with implications for sustainability. *Decision Analytics Journal*, 15, 100558.

Alhamzi, G., Javaid, S., Shuaib, U., Razaq, A., Garg, H., Razzaque, A. (2023). Enhancing interval-valued Pythagorean fuzzy decision-making through dombi-based aggregation operators. *Symmetry*, 15(3), 765.

Aliman, N.K., Mohamad, W. (2013). Perceptions of service quality and behavioral intentions: a mediation effect of patient satisfaction in the private health care in Malaysia. *International Journal of Marketing Studies*, 5, 15–29.

Alkan, N., Kahraman, C. (2025). Continuous Pythagorean fuzzy set extension with multi-attribute decision making applications. *Informatica*, 36(2), 241–283.

Alrasheedi, M.A., Jeevaraj, S. (2023). Membership score of an interval-valued Pythagorean fuzzy numbers and its applications. *IEEE Access*, 11, 37832–37839.

Alshouha, B., Serrano-Guerrero, J., Chiclana, F., Romero, F.P., Olivas, J.A. (2024). An integrated decision framework for evaluating and recommending health care services. *Applied Intelligence*, 54, 4153–4171.

Amin, M., Nasharuddin, S.Z. (2013). Hospital service quality and its effects on patient satisfaction and behavioural intention. *Clinical Governance: An International Journal*, 18(3), 238–254.

Amit, L., James, A.T., Kumar, G. (2026). Development of a framework for evaluation of quality of emergency care services of Indian hospitals. *International Journal of Health Care Quality Assurance*, 39(3), 169–186. https://doi.org/10.1108/IJHCQA-12-2024-0123.

Anjum, M., Kraiem, N., Min, H., Daradkeh, Y.I., Dutta, A.K., Shahab, S. (2025). Integrating intuitionistic fuzzy and MCDM methods for sustainable energy management in smart factories. *PLoS One*, 20(1), e0315251.

Arisoy, A.A., Jeevaraj, S., Gokasar, I., Deveci, M., Kadry, S., Liu, Z. (2025). Railway prioritized food logistics in developing countries using fuzzy decision making under interval-valued Pythagorean fuzzy environment. *Applied Soft Computing*, 175, 113066.

Atanassov, K.T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87–96.

Babroudi, N.E.P., Sabri-Laghaie, K., Ghoushchi, N.G. (2021). Re-evaluation of the healthcare service quality criteria for the Covid-19 pandemic: z-number fuzzy cognitive map. *Applied Soft Computing*, 112.

Cui, H., Tan, Q. (2025). A fuzzy decision support system for medical service quality management in hospitals. *International Journal of Computational Intelligence Systems*, 18, 57.

Darzi, M.A., Islam, S.B., Khursheed, S.O., Bhat, S.A. (2023). Service quality in the healthcare sector: a systematic review and meta-analysis. *LBS Journal of Management & Research*, 21(1), 13–29.

Datt, M., Gupta, A., Misra, S.K. (2025). Advances in management of healthcare service quality: a dual approach with model development and machine learning predictions. *Journal of Advances in Management Research*, 22(4), 675–701.

De, S.K., Biswas, R., Roy, A.R. (2000). Some operations on intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 114, 477–484.

Demir, A.T., Moslem, S. (2024). Evaluating the effect of the COVID-19 pandemic on medical waste disposal using preference selection index with CRADIS in a fuzzy environment. *Heliyon*, 10(5), e26997.

Farrokhi, P., Bagherzadeh, R., Arab-Zozani, M., Zarei, E. (2023). Assessing the quality of hospital outpatient services in Iran: a systematic review and meta-analysis. *BMC Health Services Research*, 23, 508.

Ferreira, D.C., Vieira, I., Pedro, M.I., Caldas, P., Varela, M. (2023). Patient satisfaction with healthcare services and the techniques used for its assessment: a systematic literature review and a bibliometric analysis. *Healthcare*, 11(5), 639.

Garg, H. (2016). A novel accuracy function under interval-valued Pythagorean fuzzy environment for solving multicriteria decision making problem. *Journal of Intelligent & Fuzzy Systems*, 31(1), 529–540.

Garg, H. (2017). A new improved score function of an interval-valued Pythagorean fuzzy set based TOPSIS method. *International Journal for Uncertainty Quantification*, 7(5), 463–474.

Hung, W.-L., Yang, M.-S. (2006). Fuzzy entropy on intuitionistic fuzzy sets. *International Journal of Intelligent Systems*, 21, 443–451.

Işık, Ö., Adalar, İ. (2025). A multi-criteria sustainability performance assessment based on the extended CRADIS method under intuitionistic fuzzy environment: a case study of Turkish non-life insurers. *Neural Computing and Applications*, 37, 3317–3342.

Karasan, A., Erdogan, M., Cinar, M. (2022). Healthcare service quality evaluation: an integrated decision-making methodology and a case study. *Socio-Economic Planning Sciences*, 82.

Khan, E.M., Talaie, H., Azizi, M. (2023). A healthcare service quality assessment model using a fuzzy best-worst method with application to hospitals with in-patient services. *Healthcare Analytics*, 4, 100241.

Kondasani, R.K., Panda, R.K. (2015). Customer perceived service quality, satisfaction and loyalty in Indian private healthcare. *International Journal of Health Care Quality Assurance*, 28, 452–467.

Kose, Y., Civan, H.N., Ayyildiz, E., Cevikcan, E. (2022). An interval-valued Pythagorean fuzzy AHP–TOPSIS integrated model for ergonomic assessment of setup process under SMED. *Sustainability*, 14, 13804.

Kumar, T., Bajaj, R.K., Ansari, M.D. (2020). On accuracy function and distance measures of interval-valued Pythagorean fuzzy sets with application to decision making. *Scientia Iranica E*, 27(4), 2127–2139.

Li, H., Yazdi, M., Huang, H.Z., Huang, C.-G., Peng, W., Nedjati, A., Adesina, K.A. (2023). A fuzzy rough copula Bayesian network model for solving complex hospital service quality assessment. *Complex & Intelligent Systems*, 9, 5527–5553.

Liu, A., Guo, X., Liu, T., Zhang, Y., Tsai, S.B., Zhu, Q., Hsu, C.F. (2019). A GRA-based method for evaluating medical service quality. *IEEE Access*, 7, 34252–34264.

Liu, Z., Wang, D., Deveci, M., Letchmunan, S. (2025). Interval-valued Pythagorean fuzzy distance-based extended inferior ratio method for multiattribute decision-making: application to green supplier selection in manufacturing industry. *Applied Soft Computing*, 185.

Mini, M., Olickal, J.J., Stanley, A., Anish, T.S., Chellappan, V. (2025). Perceived healthcare quality as the predictor of patient satisfaction: Findings from a public sector tertiary care hospital in Kerala, South India. *Clinical Epidemiology and Global Health*, 35, 102163.

Mishra, A.R., Rani, P., Alrasheedi, A.F., Dwivedi, R. (2023). Evaluating the blockchain-based healthcare supply chain using interval-valued Pythagorean fuzzy entropy-based decision support system. *Engineering Applications of Artificial Intelligence*, 126, 107112.

Moazzamigodarzi, M., Kumar, P., Rahimi, M. (2025). Assessing the challenges of digital transformation implementation in sustainable financial services using hybrid multi-criteria group decision-making framework. *Operational Research*, 25, 70.

Oh, Y., Pasman, H., Khan, S.A., Park, S. (2025). Hydrogen storage selection for Saudi Arabia: a multi-criteria decision making under interval-valued Pythagorean fuzzy environment. *International Journal of Hydrogen Energy*, 105, 1281–1293.

Pamucar, D., Ecer, F., Gligoric, Z., Gligoric, M., Deveci, M. (2023). A novel WENSLO and ALWAS multicriteria methodology and its application to green growth performance evaluation. *IEEE Transactions on Engineering Management*, 71, 9510–9525.

Peng, X., Li, W. (2019). Algorithms for interval-valued Pythagorean fuzzy sets in emergency decision making based on multiparametric similarity measures and WDBA. *IEEE Access*, 7, 7419–7441.

Peng, X., Yang, Y. (2016). Fundamental properties of interval-valued Pythagorean fuzzy aggregation operators. *International Journal of Fuzzy Systems*, 31, 444–487.

Puška, A., Stević, Ž., Pamučar, D. (2022). Evaluation and selection of healthcare waste incinerators using extended sustainability criteria and multi-criteria analysis methods. *Environment, Development and Sustainability*, 24, 11195–11225.

Rani, P., Alrasheedi, A.F., Mishra, A.R., Cavallaro, F. (2023). Interval-valued Pythagorean fuzzy operational competitiveness rating model for assessing the metaverse integration options of sharing economy in transportation sector. *Applied Soft Computing*, 148, 110806.

Rehman, A.U., Abidi, M.H., Usmani, Y.S., Mian, S.H., Alkhalefah, H. (2023). Development of an intuitive GUI-based fuzzy multi-criteria decision model for comprehensive hospital service quality evaluation and indexing. *Axioms*, 12(10), 921.

Saha, A., Rage, K., Senapati, T., Chatterjee, P., Zavadskas, E.K., Sliogerienė, J. (2025). A consensus-based MULTIMOORA framework under probabilistic hesitant fuzzy environment for manufacturing vendor selection. *Informatica*, 36(3), 713–736.

Sanıl, M., Eminer, F. (2021). An integrative model of patients' perceived value of healthcare service quality in North Cyprus. *Archives of Public Health*, 79, 227.

Shafii, M., Rafiei, S., Abooee, F., Bahrami, M.A., Nouhi, M., Lotfi, F., Khanjankhani, K. (2016). Assessment of service quality in teaching hospitals of Yazd university of medical sciences: using multi-criteria decision making techniques. *Osong Public Health and Research Perspectives*, 7(4), 239–247.

Sivakumar, G., Almehdawe, E., Kabir, G. (2022). Developing a decision-making framework to improve healthcare service quality during a pandemic. *Applied System Innovation*, 5(1), 3.

Wang, G., Shao, Q. (2024). Design of a smart medical service quality evaluation system based on a hybrid multi-criteria decision model. *Scientific Reports*, 14, 26407.

Yager, R.R. (2014). Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems*, 22(4), 958–965.

Yu, Z., Wang, D., Kassim, A.M., Zhu, S., Liu, Z. (2025). An extended interval-valued Pythagorean fuzzy VIKOR approach with logarithmic distance measures for service quality evaluation of university library. *Journal of Mathematics*, 2025(9966857), 1–16.

Yuan, J., Chen, Z., Wu, M. (2023). A novel distance measure and CRADIS method in picture fuzzy environment. *International Journal of Computational Intelligence Systems*, 16, 186.

Yucesan, M., Gul, M. (2020). Hospital service quality evaluation: an integrated model based on Pythagorean fuzzy AHP and fuzzy TOPSIS. *Soft Computing*, 24, 3237–3255.

Zadeh, L.A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353.

Zan, R.M., Choong, W.W., Razali, F., Amankwah, O. (2026). Evaluating the impact of healthcare facilities and service quality on patient satisfaction in Malaysian private hospitals: a PLS-SEM approach. *Property Management*, 44(4), 808–831.

Zehra, S., Ranjan, J., Shukla, M. (2025). Service quality in healthcare: understanding the relationship between patient experience and healthcare outcomes. *International Journal of Health Care Quality Assurance*, 38(4), 233–250.

Zhang, X. (2016). Multicriteria Pythagorean fuzzy decision analysis: a hierarchical QUALIFLEX approach with the closeness index-based ranking methods. *Information Sciences*, 330, 104–124.

A. Almuflih is an associate professor in industrial engineering at King Khalid University. His research interests lie in the intersection of operations management and data analytics, in particular, as applied to healthcare delivery system framework, supply chain engineering and lean concepts. He completed his PhD in the Department of Industrial Engineering and Management System at University of Central Florida (UCF) in 2020. He also completed a MS degree in the Department of Mechanical and Industrial Engineering at Northeastern University (NEU) in 2016 and BS at KKU in 2011.

A. Al Fatais is an assistant professor in the Industrial Engineering Department at King Khalid University (KKU). He received his BS in industrial engineering from KKU, and MS in engineering management and leadership from Santa Clara University (SCU). In addition, Dr. Al Fatais received his PhD in industrial engineering from UCF. His research interests include advanced manufacturing technologies, artificial intelligence, and operation management.

A.R. Mishra received his PhD in mathematics and he is an assistant professor in Department of Mathematics, Government Vivekanand College Maihar, Madhya Pradesh, India. His main research interests are fuzzy sets theory, decision theory, multi-criteria decision making, intuitionistic fuzzy sets, orthopair fuzzy sets, neutrosophic fuzzy sets, picture fuzzy sets, hesitant fuzzy sets, rough sets information measures, aggregation operators, soft computing models and others. He has published more than 200 peer-reviewed papers, many in high-quality international journals. According to Stanford University, he is among world's 2% scientists in the field of Artificial Intelligence since 2021.

E.B. Tirkolaee obtained a BSc (2012) and an MSc (2014) in industrial engineering from Isfahan University of Technology, Iran. He then received a PhD (2019) in industrial engineering from Mazandaran University of Science and Technology, Iran. He is currently an associate professor in the Department of Industrial Engineering at Istinye University in Istanbul, Turkey. He has published more than 200 papers in high-quality journals. Dr. E.B. Tirkolaee is currently an editor in different prestigious WoS/Scopus journals, such as *IEEE Transactions of Fuzzy Systems* (*IEEE*), *Artificial Intelligence Review* (Springer), and *Carbon Balance and Management* (Springer). His main research interests include operations research, supply chain management, fuzzy systems, and multi-criteria decision-making. Dr. E.B. Tirkolaee has been featured on the "World's Top 2% Researchers/Scientists List (Career-Long & Single-Year)" identified by Elsevier BV-Stanford University since 2021.