

# Method ZAPROS for Multicriteria Alternatives Ranking and the Problem of Incomparability

Oleg LARICHEV

*Institute for Systems Analysis  
60 let Octjabrja 9, 117312 Moscow, Russia  
e-mail: oil@isa.ru*

**Abstract.** The new method for the construction of partial order on the set of multicriteria alternatives is presented. This method belongs to the family of Verbal Decision Analysis methods and gives a more efficient means of problem solution. The method is based on psychologically valid operations for information elicitation from a decision maker: comparisons of two distances between the evaluations on the ordinal scales of two criteria. The information received from a decision maker is used for the construction of a binary relation between a pair of alternatives which yields preference, indifference and incomparability relations. The notion of a method decisive power is introduced. The illustrative example is given.

**Key words:** decision making, multiple criteria evaluation, verbal decision analysis, qualitative estimates, ZAPROS method.

## 1. Introduction

The unstructured decision problems (Newell and Simon, 1958) are widespread in practice. Let us stress the common features of such problems:

1. Factors in these problems are of purely qualitative, subjective nature, especially difficult for formalization and numerical measurement (prestige of an organization, attractiveness of a dress, attitude towards reforms, etc.); the factors are usually described in language accepted by the decision maker;
2. The process of task analysis is also subjective by nature: rules for consideration and comparison of the main qualitative factors are mainly defined by the decision maker.

Therefore, the decision maker is the key element of the problem. This must be recognized, and attention must be paid to the capabilities and limitations of the human information processing system and to the results of investigations on human errors and heuristics (Kahneman *et al.*, 1982).

In this paper we discuss the multicriteria ranking problem having the following features:

- 1) A decision rule is to be developed before appearance of alternatives;
- 2) There are a large number of alternatives;

- 3) Evaluations of alternatives upon criteria could be given only by human beings playing the role of measurement devices.
- 4) The quality grades on criteria scales are verbal definitions presenting subjective values of the decision maker.

The method ZAPROS (abbreviation of Russian words: Closed Procedures near Reference Situations) has been developed for the solution of problems having such features. This method belongs to the family of Verbal Decision Analysis (VDA) methods (Larichev and Moshkovich, 1977).

The method ZAPROS-III presented in this paper uses the preference elicitation procedure proposed in first version of the method (Larichev *et al.*, 1978). In the difference from ZAPROS-LM method (Larichev and Moshkovich, 1995), the comparison of quality variations along criteria scales (instead of criteria estimates comparison) is used. For the first time in this paper the notion of the method “decisive power” is introduced as well the technique of its calculation. Additionally to it, the procedure of a decision rule elaboration has quite a different structure to give more rational and strict justification to the method:

1. A simpler and more transparent procedure for the construction of a joint ordinal scale for quality variation along the scales of criteria is used.
2. New justification is given for the procedure of alternatives comparison

## 2. Example

The practical problem is to organize a fund for investing money in R&D projects. The fund organizer is interested in developing an effective system for selection of the best projects. A decision analyst is used to carry out the job. It is decided that highly qualified experts are to be involved in the process of project estimation.

The fund organizer (we will call him the decision-maker) in cooperation with the analyst develop a list of the most important criteria for project evaluation. The list of these criteria with possible values on their scales is presented in Table 1.

Table 1  
Criteria and possible values for evaluation of R&D projects

Criteria	Possible values on their scales
A. Originality	A1. Absolutely new idea and/or approach A2. There are new elements in the proposal A3. Further development of previous ideas
B. Prospects	B1. High probability of success B2. Success is rather probable B3. Success is hardly probable
C. Qualification	C1. Qualification of the applicant is high C2. Qualification of the applicant is normal C3. Qualification of the applicant is unknown

It is easy to note that criterion values are given from the most preferred to the least preferred (according to the preferences of the decision maker).

The projects to be submitted to the fund are not known in advance. It is necessary to rank-order the submitted projects according to their overall value. Each project requires some resources. Given a ranking of projects it is easy to select a group of best projects within the limit of available resources.

The question is: how to construct a rank-order for all possible combinations of the evaluations upon the criteria (in our case 27 combinations) on the basis of the decision maker preferences?

### 3. Formal Statement of Problem

The problem may be formulated as follows.

*Given:*

1.  $K = 1, 2, \dots, N$  is a set of criteria;
2.  $n_q$  is the number of possible values on the scale of the  $q$ -th criterion ( $q \in K$ );
3.  $X_q = \{x_{iq}\}$  is a set of values for the  $q$ -th criterion (the scale of the  $q$ -th criterion);  $|X_q| = n_q$  ( $q \in K$ ); the values on a scale are ordered from best (first) to worst (last); the order of the values on one scale does not depend upon values on the others;
4.  $Y = X_1 * X_2 * \dots * X_N$  is a set of vectors  $y_i \in Y$  of the following type  

$$y_i = (y_{i1}, y_{i2}, \dots, y_{iN}), \text{ where } y_{iq} \in X_q \text{ and } P = |Y| = \prod_{i=1}^{i=N} n_i;$$
5.  $A = \{a_i\} \in Y; i = 1, 2, \dots, t$  – the set of  $t$  vectors describing real alternatives.

*Required:*

to rank multicriteria alternatives on the basis of a decision-maker's preferences.

### 4. Elicitation of DM's Preferences

#### 4.1. Joint Scale of Quality Variation for Two Criteria

Let us look at the criteria list and assume that we have an "ideal" object, assigned all the best values on all criteria. We usually do not have such an alternative in real life. We will use this ideal alternative as a "reference situation". Deviating from this ideal, we will lessen the quality of the hypothetical object on two criteria.

Let us introduce the notion of quality variation (QV). Quality variation is the result of changing one evaluation on the scale of one criteria.

The task of decision maker preference elicitation consists in pair-wise comparison of all QV taken from the scales of two criteria, by supposition that there are the best

evaluations on the other criteria. Evidently, QV along a criterion scale is equal to the sum of QV between the evaluations on the same scale.

The typical question posed to the decision maker is:

“What do you prefer for the transfers from better evaluations to worse ones:

$$x_{if} \Rightarrow x_{ik} \quad \text{or} \quad x_{js} \Rightarrow x_{ju} \quad (k > f; u > s)?”$$

The possible decision maker answers are: «the first» or «the second» or «they are equal for me».

Decision maker responses allow ranking of all QV from the scales of two criteria. This ranking could be called the Joint Scale of Quality Variation (JSQV) for two criteria.

Next, a different pair of criteria is taken for QV comparison by supposition that the evaluations for other are best.

There are  $0.5N(N - 1)$  possible pairs of criteria. The preferences of the decision maker are elicited for each pair. So,  $0.5N(N - 1)$  rankings of QV for all pairs of criteria could be constructed.

Let us illustrate the method by the example.

Let us look at the criteria list and assume that we have an «ideal» project, assigned all the best values on all criteria. We will lessen the quality of the hypothetical project against two criteria: A («Originality») and B («Prospects») criteria having the best evaluations on C. In the example we have three QV for each criterion. Therefore, the task for the decision maker consists in pair-wise comparison of six QV. Not all comparisons are needed: QV along the scales are equal to the sum of QV between the evaluations on the same scale. But, it is possible to use the transitive closure: for example, if

$$b_3 > b_2 \quad \text{and} \quad b_2 > a_2 \quad \text{than} \quad b_3 > a_2.$$

Eight comparisons are needed to rank QV's from the scales of A and B (4 relations follow from transitivity).

Let us introduce the following notions for QV's:

$$\begin{aligned} A_1 \Rightarrow A_2 = a_1; \quad A_2 \Rightarrow A_3 = a_2; \quad A_1 \Rightarrow A_3 = a_3, \\ B_1 \Rightarrow B_2 = b_1; \quad B_2 \Rightarrow B_3 = b_2; \quad B_1 \Rightarrow B_3 = b_3. \end{aligned}$$

The generic question to DM is:

*Question.* “What do you prefer: the transfer from a project with an absolutely new idea to one with new elements in the proposal or the transfer from a project but with high probability of success to one where the success is rather probable?”

Posing the questions in a similar way one could rank QV. Let us suppose that DM preferences define the following ranking:

$$a_1 \langle b_1 \langle a_2 \langle b_2 \langle a_3 \langle b_3$$

This ranking is JSQV for the criteria A and B by presupposition that it is the best evaluation on criterion C.

In the same way JSQV for the pairs (A and C) and (B and C) can be constructed.

#### 4.2. The Check of Independence for Two Criteria

JSQV for a pair of criteria holds valuable information about DM preferences. But the possibility of its utilization depends on the independence of the decision maker comparisons with a “reference situation”.

DEFINITION 1. Let us call two criteria *independent by quality variation* if JSQV constructed for those criteria do not depend from the evaluations on other criteria.

The check of independence by QV is.

Let us take a quite different “reference situation” – the worst evaluations on all criteria. It is possible to compare QV on the scale of two criteria and check the correspondence to the ranking made near first “reference situation”.

Only some part of QV comparisons could be taken with the aim of testing the independence condition. If there is no difference in QV comparisons for the same pair of criteria near different “reference situations” we could accept that the two criteria are independent by QV.

Two “reference situations” are quite contrasting alternatives. It is possible to accept that the condition of independence is true if such “reference situations” have no influence on the comparisons made by the decision maker.

For the example given above it is necessary to repeat some comparisons of QV by supposition about the evaluation C3 on criterion C.

Let us suppose that the results of the comparisons are:

$$b_3 > a_3 > b_2 > a_2,$$
$$b_3 > a_3 > b_2 > a_2.$$

In this case we could accept the independence of criteria A and B.

For cases when a pair of criteria are not independent by QV, it is necessary to change the verbal description of a problem and achieve independence (see the examples in (Larichev and Moshkovich, 1997)).

Let us note that the condition of independence by QV is close to the condition of preference independence (Keeney and Raiffa, 1976).

#### 4.3. Independence for the Group of Criteria

The dependence of a pair of criteria on the rest of the criteria is the best-understood case. It might be well to note that this kind of (in)dependence of criteria is checked in many decision methods. It was proven that, if all criteria are pair-wise preference independent, any group of criteria is independent of the rest of criteria (Keeney and Raiffa, 1976).

We refer to the opinion of Winterfeldt and Fisher (1975) that the group dependence of criteria «is indefinite in nature and difficult to detect» if the criteria are pair-wise independent. Really, one could not find such dependence in practical cases.

### *Consequence*

In the case when all pairs of criteria are independent by QV it is possible to accept that all criteria are independent by QV.

The test for independence is sufficient because all pairs of criteria are considered. Therefore, one could take any pair of criteria independently from the others to analyze the differences in evaluations of alternatives.

#### 4.4. *Joint Scale of Quality Variation for all Criteria*

On the basis of information elicited from the decision maker for each pair of criteria it is possible to construct JSQV for all criteria. The noncontradictory rankings of QV from all criteria scales are compared many times. It is necessary to find the place of each QV on the joint scale.

For the construction of the joint scale for all criteria it is possible to use the following algorithm.

#### *Sequential selection of non-dominating QV*

Joint Scales for the pairs of criteria could be taken as the graphs having the same root: zero quality decreasing. Let us take the node of this joint graph that is not dominated by any other and put it on the joint scale. After excluding such nodes from all graphs, let us find the next non-dominated node, put it on the joint scale and so on. It is easy to see that such algorithm gives the rank-order of all QV.

Let us take the example given above.

Additional to the JSQV given above for criteria A and B, it is possible to construct scales of such kind for the pairs (A and C) and (B and C). Let us suppose that we have the following results.

$$c_1 \langle a_1 \langle a_2 \langle c_2 \langle a_3 \langle c_3 \quad \text{and} \quad c_1 \langle b_1 \langle b_2 \langle c_2 \langle b_3 \langle c_3.$$

Using the algorithm of sequential selection of non-dominated QV, it is possible to construct the joint scale of QV for all criteria:

$$c_1 \langle a_1 \langle b_1 \langle a_2 \langle b_2 \langle c_2 \langle a_3 \langle b_3 \langle c_3.$$

#### 4.5. *The Check of Information for Contradictions*

In the process of construction of the JSQV for two criteria, it is easy for the decision maker to check comparisons for possible contradictions. That is not the case for the construction of JSQV for all criteria.

Certainly, people could make errors. Therefore, we need special procedures for finding and eliminating human errors.

Fortunately, a special, "closed" procedure for finding and eliminating the decision maker contradictions has been proposed (Larichev *et al.*, 1978).

By constructing JSQV for every pair of criteria we would require additional data from the decision maker. The additional information is used to create the check for consistency.

If on some step of the algorithm for sequential selection of QV it is not possible to find the next non-dominated QV, there is a contradiction in the decision maker preferences. The algorithm can discover the contradictory answer and demonstrate them to the decision maker for correction.

Let us return to the example presented above.

Let us suppose that instead of JSQV for criteria C and B given above, we have the following:

$$b_1 \prec c_1 \prec b_2 \prec c_2 \prec b_3 \prec c_3.$$

In this case, when combining three scales into one, there is the contradiction:

$$b_1 \prec c_1 \prec a_1 \prec b_1.$$

The contradiction does not allow finding a place of corresponding QV on the joint scale. Usually such a contradiction is the result of an irrational judgment. It is necessary for the decision maker to analyse the situation and find a rational compromise.

This contradiction is to be presented to the decision maker for analysis and resolution.

The construction of Joint Scale of Criteria Variations gives a check of the decision maker input for contradiction. The possibility to combine pair-wise scale into a JSCV confirms the absence of contradictions in decision maker judgments.

The questions needed for JSCV construction represent the dialog between the decision maker and the computer.

#### 4.6. *Psychological Basis of the Procedure*

The procedure of decision maker preference elicitation proposed above is justified from a psychological point of view. All questions to the decision maker are formulated in natural language, in terms of verbal evaluations on criteria scales. The kind of questions (comparison of two quality variations) is admissible (Larichev, 1992). The psychological studies show that the decision maker is capable of performing such operation with small inconsistencies and using complex strategies (Larichev, 1992; Larichev and Moshkovich, 1997).

The proposed procedure of DM preference elicitation was checked in experiments with a group of subjects (Larichev *et al.*, 1978). For 5 to 7 criteria with 2 to 5 evaluations on criteria scales the number of contradictions was 1 to 3. In the average subjects make one or two contradictions answering 50 questions. When subjects were presented the contradictions, they removed them to construct a consistent decision rule.

### 5. Comparison of Alternatives

**Statement 1.** *The quality of every alternative can be expressed as the vector of QV corresponding to the evaluations of the alternative upon the criteria.*

*Proof.* Each evaluation of an alternative is connected with some QV. In the case of independence by quality variation, it is possible to represent the quality of an alternative by the set of QV, each of them corresponds to the distance along a scale of one criterion between the evaluations. Therefore, the vector QV represents the quality of an alternative.

**Statement 2.** *The relation between any pair of QV on JSQV is defined or determined by direct answers of DM or on the basis of expansion by transitivity.*

*Proof.* Let us take two arbitrary QV from JSQV. It is possible to find JSQV for the pair of criteria or for one criterion that they belong to. For both cases those two QV are compared or assessed directly by the decision maker or by utilization of the transitivity condition.

DEFINITION 2. Let us note as the function of alternative quality:  $V(y)$ .

Let us make the following supposition about the properties of this function:

- there are maximum and minimum values of  $V(y)$ ;
- for independent criteria, the value of  $V(y)$  is increasing when the evaluation on each criteria are improving.

Let us assign a rank for each QV on JSQV beginning from the best QV.

For example, for JSQV given above,

$$c_1 \prec a_1 \prec b_1 \prec a_2 \prec b_2 \prec c_2 \prec a_3 \prec b_3 \prec c_3.$$

rank 1 is given to  $c_1$ , rank 2 is given to  $a_1$  and so on.

Let us take two alternatives:  $y_i = (y_{i1}, y_{i2}, \dots, y_{iN})$  and  $y_j = (y_{j1}, y_{j2}, \dots, y_{jN})$ .

It is possible to find a corresponding QV for each component of vectors and rank each QV according to JSQV.

For each alternative it is possible to define the corresponding vector of components ranks

$$\begin{aligned} V(y_i) &\Leftrightarrow V(r_k, r_l, \dots, r_g), \\ V(y_j) &\Leftrightarrow V(q_s, q_d, \dots, q_m), \end{aligned}$$

where

$$\begin{aligned} r_k, r_l, \dots, r_g &- \text{ranks of components for the vector } y_i = (y_{i1}, y_{i2}, \dots, y_{iN}); \\ q_s, q_d, \dots, q_m &- \text{ranks of components for the vector } y_j = (y_{j1}, y_{j2}, \dots, y_{jN}). \end{aligned}$$

**Statement 3.** *If the condition of independence by QV is true for all pairs of criteria and ranks of the components for  $y_i$  are no worse than the ranks of the components for  $y_j$  and at least for one components of  $y_i$  rank is better, than alternative  $y_i$  is more preferable for the decision maker in the comparison with  $y_j$ , and  $V(y_i) \succ V(y_j)$ .*



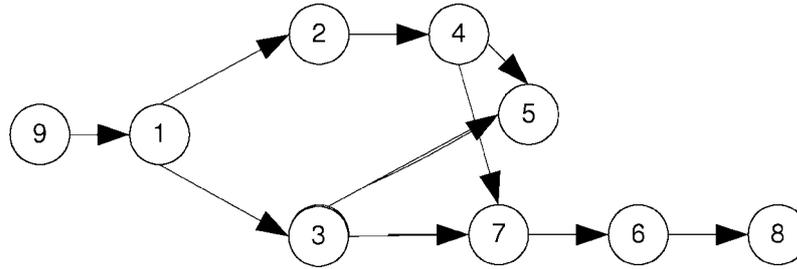


Fig. 1. The partial order of alternatives.

Let us demonstrate how the comparisons are made. First, the comparison of Alt 9 and Alt 1 is done. It corresponds to the comparison of  $a_1$  and  $b_1$  on general JSQV. According to JSQV (see above) Alt 9 is better.

Then, Alt 1 is compared with 2 and 3. It is not necessary to compare Alt 9 with Alt 2 and Alt 3 because they are dominated by Alt 1. Really,  $c_1$  is less than  $a_1$ , and  $b_1$  is less than  $a_3$  on general JSQV. The next alternative involved in the process of the comparison is Alt 4 and so on. Let us take Alt 5 and 7. In pair-wise comparison one needs to compare  $c_3$  with  $a_2$  and  $b_1$ . We do not have enough information on JSQV to make such a comparison.

Let us note that 15 binary comparisons are made to obtain the partial order of the alternatives presented on Fig. 1.

## 6. Decisive Power of ZAPROS Method

The utilization of incomparability relation puts the question on how decisive a decision method is. In other words, it is valuable to know how often alternatives could be in incomparability relation.

First time the question was discussed in (Larichev, 1997). The evaluation of decisive power of ZAPROS was made for the case of binary scales. It was found that in general case the number of alternatives pair with incomparability relation is no more than 10% from Cartesian product of criteria scales (full set of alternatives). The result was obtained in analytic form.

In the general case (scales with different number of estimates) the evaluation of a normative method decisive power could be obtained by modeling. Using fast computers it is possible to create all alternative pairs and find ones that could not be compared by ZAPROS (on the basis of JSQV). The result of the calculations depends evidently from JSQV.

Let us demonstrate how the estimation of ZAPROS III decisive power could be made. Let  $Q$  be the general number of alternative pairs:

$$Q = 0.5n^N(n^N - 1).$$

Among them there is subset from  $D$  pairs that are always in Pareto dominance relation.

The difference between  $Q$  and  $D$  create alternative pairs having the following property: relation between the alternatives depends from DM preferences. Let us call such pairs as potentially contradictory alternative pairs (PCA pairs).

Now we could formally introduce the index of decisive power of a normative method

$$P = 1 - \frac{S}{B},$$

where  $B = Q - D$  and  $S$  is the number of alternatives that could not be compared on the basis of JSQV.

Let us take the example given above with the following scale of Quality Variation:

$$c_1 \langle a_1 \langle b_1 \langle a_2 \langle b_2 \langle c_2 \langle a_3 \langle b_3 \langle c_3.$$

Let  $N = 3$  and  $n = 3$ .

We have:  $Q = 351$ ;  $D = 189$ ;  $B = 162$ ;  $S = 27$  and  $P = 0.942$ .

## **7. Conclusion**

Together with the big variety of multicriteria problems there are many methods for the solution (Keeney and Raiffa, 1976; Roy, 1996; Saaty, 1980). The approach of Verbal Decision Analysis (VDA) is oriented to the solution of unstructured problems (Larichev, 1992; Larichev and Moshkovich, 1997).

The important feature of VDA methods is the utilization of psychologically justified ways of a decision maker preference elicitation. Such an approach takes into account the possibilities and limitations of human information processing system. VDA methods are based on the utilization of natural language on every step of the analysis. The methods do not require from an user any preliminary knowledge about decision analysis.

In a comparison with MAUT approach the output of ZAPROS is very approximate. Some alternatives could be incomparable. Alternatives have only ranks (sometimes fuzzy) instead of exact quantitative evaluations of utility.

But such approximate output is much more reliable. A decision maker could use ZAPROS to gradually develop a consistent and non-contradictory policy. In experiments (Larichev *et al.*, 1995) it was demonstrated that the methods based on MAUT are very sensitive to small human errors. Such errors are inevitable because human beings are not exact measurement devices producing exact quantitative measurements.

On the other hand, the relations between alternatives received as the output of ZAPROS are stable. For many practical problems the method decisive power is big enough.

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## References

- Kahneman, D., P. Slovic, A. Tversky (Eds.) (1982). *Judgement Under Uncertainty: Heuristics And Biases*. Cambridge University Press, Cambridge.
- Keeney, R.L., H. Raiffa (1976) *Decisions With Multiple Objectives: Preferences and Value Tradeoffs*. Wiley, New York.
- Larichev, O.I., Yu.A. Zuev, L.S. Gnedenko (1978). Method ZAPROS (CLosed Procedures near Reference Situations) for the analysis of variants of complex decisions. In: S.V. Emelyanov (Ed.), *Multicriteria Choice for the Solution of Ill-structured Problems*, Moscow: VNIISI Proceedings, **5**, pp. 83–97 (in Russian).
- Larichev, O.I. (1992). Cognitive validity in design of decision-aiding techniques. *Journal of Multi-Criteria Decision Analysis*, **1**(3), 127–138.
- Larichev, O.I., H.M. Moshkovich (1995). ZAPROS-LM – a method and system for rank-ordering of multiattribute alternatives. *European Journal of Operations Research*, **82**, 503–521.
- Larichev, O.I., D.L. Olson, H.M. Moshkovich, A.I. Mechitov (1995). Numerical Vs. cardinal measurements in multiattribute decision making: how exact is exact enough? *Organizational Behavior And Human Decision Processes*, **64**(1), 9–21.
- Larichev, O. (1997). Measurement of differences in preferences expressed by a simple additive rule. In M. Karwan, J. Spronk, J. Wallenius (Eds.), *Essays in Decision Making*. Springer Verlag, Berlin. pp. 168–184.
- Larichev, O.I., H.M. Moshkovich (1997). *Verbal Decision Analysis For Unstructured Problems*. Kluwer Academic Publishers, Boston, 258pp.
- Newell, A., H. Simon (1958). Heuristic Problem solving: the next advance in operations research. *Operations Research*, **6**, 1–10.
- Roy, B. (1996). *Multicriteria Methodology for Decision Aiding*. Kluwer Academic Publisher, Dordrecht, 288pp.
- Saaty, T.L. (1980). *The Analytic Hierarchy Process*. New York, McGraw-Hill.
- von Winterfeldt, D., G.W. Fischer (1975). Multiattribute utility theory: models and assessment procedures. In: D. Wendt, C. Vlek (Eds.), *Utility, Probability and Human Decision Making*, Reidel, Dordrecht. pp. 47–86.

**O. Larichev** is a professor, Dr., a head of department at Institute for Systems Analysis, Russian Academy of Sciences. His research interests include multicriteria decision making; multicriteria mathematical programming; psychology of decision making; artificial intelligence, expert knowledge acquisition; computer tutoring systems; systems analysis.

## Daugiakriterinių alternatyvų rūšiavimo metodas ZAPROS ir nepalyginamumo problema

Oleg LARICHEV

Straipsnyje pateikiamas naujas metodas dalinei tvarkai daugiakriterinių alternatyvų aibėje sukonstruoti. Šis metodas priklauso Verbalinės Sprendimų Analizės metodų šeimai ir teikia priemones efektyvesniam problemos sprendimui. Metodas grindžiamas sprendimų priėmėjo (SP) požiūriu psichologiškai priimtinais operacijomis informacijai tikslinti, palyginant dviejų kriterijų įverčius ranginėje skalėje. Informacija, gauta iš SP, yra panaudojama sukonstruoti binarinį ryšį tarp alternatyvų porų, o tai leidžia įvesti pranašumo, indiferencijos ir nepalyginamumo santykius. Įvedama metodo sprendžiamosios galios sąvoka. Metodas iliustruojamas pavyzdžiu.