INFORMATICA, 2008, Vol. 19, No. 4, 555–566

Robust Watermarking based on Subsampling and Nonnegative Matrix Factorization *

Wei LU[1,2], Hongtao LU[2]

[1]*Department of Computer Science and*
Guangdong Key Laboratory of Information Security Technology
Sun Yat-sen University, Guangzhou 510275, China

[2]*Department of Computer Science and Engineering*
Shanghai Jiao Tong University, Shanghai 200240, China
e-mail: luwei3@mail.sysu.edu.cn, lu-ht@cs.sjtu.edu.cn

Received: August 2007; accepted: May 2008

Abstract. This paper presents a novel robust digital image watermarking scheme using subsampling and nonnegative matrix factorization. Firstly, subsampling is used to construct a subimage sequence. Then, based on the column similarity of the subimage sequence, nonnegative matrix factorization (NMF) is applied to decompose the sequence. A Gaussian pseudo-random watermark sequence is embedded in the factorized decomposition coefficients. Because of the high similarity of subimages and meaningful factorization for NMF, the proposed scheme can achieve good robustness, especially to common permutation attacks. Numerical experiment results demonstrate the good performance of the proposed scheme.

Keywords: watermarking, nonnegative matrix factorization, DWT.

1. Introduction

In the past two decades, digital watermarking technology has been devoted significantly and applied in digital right protection and authentications widely (Chang and Tseng, 2004; Hassanien, 2007; Chang and Chang, 2007). On one hand, many different approaches are introduced into digital watermarking, such as signal processing, pattern recognition, communication theory etc., which have improved the performance of watermarking. On the other hand, most of the current watermarking schemes are still not applicable in practice, since the performance of these watermarking algorithms are still far from practical application, especially for robustness, which is the most important design criteria.

Recently, a new signal decomposition method is proposed as nonnegative matrix factorization (NMF) (Lee and Seung, 1999; Lee and Seung, 2001), which decomposes a nonnegative matrix into two physically meaningful nonnegative matrices, and has been

*This work was supported by NSFC (No. 60803136, No. 60573033), the Scientific Research Foundation for the Young Teachers in Sun Yat-sen University, and Program for New Century Excellent Talents in Uniersuty (No. NCET-05-0397).

successfully applied in many signal analysis domains (Berrya *et al.*, 2006), such as frontal face verification (Zafeiriou *et al.*, 2006), blind signal separation (Cichocki *et al.*, 2006), image classification (Guillamet *et al.*, 2003). To our current knowledge, there are only a few applications for NMF in digital watermarking (Ghaderpanah and Hamza, 2006). In this paper, we propose a novel digital image watermarking scheme using NMF in subsampling domain. Through embedding a pseudo-random sequence in the NMF coefficient matrix, the proposed scheme can resist many signal attacks and distortions, and thus achieves strong robustness.

The rest parts of this paper are organized as follows. In Section 2, we introduce the subsampling technique. In Section 3, we introduce NMF algorithm briefly. In Section 4, the prosed watermarking scheme is described in detail. Then, the experimental results are shown in Section 5. Finally, conclusions are given in Section 6.

2. Subsampling

Subsampling is a common method in signal processing and analysis. For a digital image I with size $M \times N$, subsampling decomposes it into mn subimages of size $M/m \times N/n$, where m and n are the subsampling intervals on the direction of column and row, which can be described as follows:

$$\begin{aligned}
&I_1(i,j) = I(m*(i-1)+1, n*(j-1)+1),\\
&I_2(i,j) = I(m*(i-1)+1, n*(j-1)+2),\\
&I_3(i,j) = I(m*(i-1)+1, n*(j-1)+3),\\
&\quad\cdots\cdots\\
&I_{n+1}(i,j) = I(m*(i-1)+2, n*(j-1)+1),\\
&I_{n+2}(i,j) = I(m*(i-1)+2, n*(j-1)+2),\\
&\quad\cdots\cdots\\
&I_{mn}(i,j) = I(m*i, n*j),
\end{aligned} \tag{1}$$

where $I_1, I_2, \cdots, I_{mn}$ denote the subimages. Generally, there are high similarity among these submiages, i.e., the correlation coefficients between any two subimages are approximately equal to 1, $\rho(I_i, I_j) \approx 1, i, j = 1, 2, \cdots, mn$. Fig. 1 shows some subimage examples, we can see that these subimages are highly similar. Based on this distinct character, subsampling has been used in watermarking (Chu, 2003; Lu *et al.*, 2006). In (Chu, 2003), a subsampling based watermarking is proposed, which can achieve better robustness than Cox's scheme (Cox *et al.*, 1997). However, there is a serious weakness for subsampling technique in this scheme. We have proposed a permutation attack to it and defeat the scheme simply and effectively (Lu *et al.*, 2005). Then, a more robust anti-permutation watermarking was developed in (Lo *et al.*, 2007).

Fig. 1. The first four subimages from Lenna (512×512), where the subsampling parameter $m, n = 4$.

3. Nonnegative Matrix Factorization

Nonnegative matrix factorization (NMF) is developed as a matrix factorization technique, which decomposes nonnegative matrices into physically meaningful data in two dimensional signal analysis, and has been used for image representation, document analysis and clustering for its parts-based representation property. NMF results in a reduced representation of the original data. Thus, NMF can also be a feature extraction or a dimensionality reduction technique. A formal description of nonnegative matrix factorization can be described as follows. Given a nonnegative matrix $I \in \mathbb{R}^{m\times n}$ and a positive integer $p < \min(m, n)$, NMF aims to find nonnegative matrix $W \in \mathbb{R}^{m\times p}$ and $H \in \mathbb{R}^{p\times n}$ to minimize the function

$$f(W, H) = \frac{1}{2}\|I - WH\|_F^2. \tag{2}$$

The product WH is called a NMF of I, where W is the normalized factor vectors by columns, H is the encoding vectors. In other words, NMF decomposes a nonnegative matrix as follows:

$$I \approx WH. \tag{3}$$

Here, we briefly describe the multiplicative iteration algorithm of NMF proposed in (Lee and Seung, 1999). Firstly, an objective function is selected based on the Poisson

likelihood as follows:

$$D(I, WH) = \sum_{i=1}^{m} \sum_{j=1}^{n} \Big(I_{ij} \ln \frac{I_{ij}}{(WH)_{ij}} - I_{ij} + (WH)_{ij} \Big), \tag{4}$$

which, then, is simplified through some eliminiation of pure date terms, and we obtain

$$D(I, WH) = \sum_{i=1}^{m} \sum_{j=1}^{n} \Big(\sum_{k=1}^{p} W_{ik} H_{kj} - I_{ij} \ln \sum_{k=1}^{p} W_{ik} H_{kj} \Big). \tag{5}$$

Taking the derivative with respect to H, we have

$$\frac{\partial}{\partial H_{ab}} D(I, WH) = \sum_{i=1}^{p} W_{ia} - \sum_{i=1}^{p} \frac{I_{ib} W_{ia}}{\sum_{k=1}^{n} W_{ik} H_{kb}}. \tag{6}$$

The gradient algorithm then states:

$$H_{ab} \leftarrow H_{ab} - \eta_{ab} \frac{\partial}{\partial H_{ab}} D(I, WH), \tag{7}$$

$$H_{ab} \leftarrow H_{ab} - \eta_{ab} \Big[\sum_{i=1}^{p} \frac{I_{ib} W_{ia}}{\sum_{k=1}^{q} W_{ik} H_{kb}} - \sum_{i=1}^{p} W_{ia} \Big] \tag{8}$$

for some step size η_{ab}. Forcing

$$\eta_{ab} = \frac{H_{ab}}{\sum_{i=1}^{p} W_{ia}} \tag{9}$$

gives the multiplicative rule:

$$H_{ab} \leftarrow H_{ab} \frac{\sum_{i=1}^{p} (W_{ia} I_{ib}) / \sum_{k=1}^{q} W_{ik} H_{kb}}{\sum_{i=1}^{p} W_{ia}}. \tag{10}$$

Taking the derivative with respect to W gives

$$\frac{\partial}{\partial W_{cd}} D(I, WH) = \sum_{j=1}^{n} H_{dj} - \sum_{j=1}^{n} \frac{I_{cj} H_{dj}}{\sum_{k=1}^{q} W_{ck} H_{kj}}. \tag{11}$$

The gradient algorithm then states:

$$W_{cd} \leftarrow W_{cd} - \nu_{cd} \frac{\partial}{\partial W_{cd}} D(I, WH), \tag{12}$$

$$W_{cd} \leftarrow W_{cd} - \nu_{cd} \Big[\sum_{j=1}^{n} I_{cj} \frac{H_{dj}}{\sum_{k=1}^{q} W_{ck} H_{kj}} - \sum_{j=1}^{n} H_{dj} \Big]. \tag{13}$$

Forceing the step size

$$\nu_{cd} = \frac{W_{cd}}{\sum_{j=1}^{n} H_{dj}} \tag{14}$$

gives

$$W_{cd} \leftarrow W_{cd} \frac{\sum_{j=1}^{n} (H_{dj} I_{cj}) / \sum_{k=1}^{q} W_{ck} H_{kj}}{\sum_{j=1}^{n} H_{dj}}. \tag{15}$$

The iterative computation will stop until the objective function converges. The detailed description of NMF algorithm can be found in (Pascual–Montano *et al.*, 2006; Berrya *et al.*, 2006). The computational procedure is given as follows:

1. Initialize W and H with positive random numbers.
2. For each basis vector $W_a \in R^{m \times 1}$, update the corresponding encoding vector $H_a \in R^{1 \times n}$, followed by updating and normalizing the basis vector W_a. Repeat this process until convergence.

Formally, the iteration algorithm in step 2 is as follows:

Algorithm 1 NMF algorithm

procedure NONNEGATIVE MATRIX FACTORIZATION I
  while not convergence **do**
    for $a = 1, 2, \cdots, m$ **do**
      for $b = 1, 2, \cdots, n$ **do**
        $H_{ab} \leftarrow H_{ab} \frac{\sum_{i=1}^{p} (W_{ia} I_{ib}) / \sum_{k=1}^{q} W_{ik} H_{kb}}{\sum_{i=1}^{p} W_{ia}}$
        for $c = 1, 2, \cdots, p$ **do**
          $W_{cd} \leftarrow W_{cd} \frac{\sum_{j=1}^{n} (H_{dj} I_{cj}) / \sum_{k=1}^{q} W_{ck} H_{kj}}{\sum_{j=1}^{n} H_{dj}}$
          $W_{ca} \leftarrow \frac{W_{ca}}{\sum_{j=1}^{n} W_{ja}}$
        end for
      end for
    end for
  end while
end procedure

4. Watermarking Procedure

In this section, we introduce a novel image watermarking scheme using subsampling and NMF. The detail algorithm is as follows.

4.1. *Watermark Embedding*

In our proposed scheme, the watermark is a pseudo-random Gaussian sequence M with length l, i.e., $M = w_1 w_2 \cdots w_l$, which is embedded in a given image I. An illustration for the watermark embedding process is shown in Fig. 2. Firstly, the image I is decomposed using subsampling equation (1) into mn subimages, i.e., $I_1, I_2, \cdots, I_{mn}$. For each subimage I_i with size $M/m \times N/n$, it is spread to a column vector in a zigzag order:

$$\begin{aligned}
&I_i^c(1) = I_i(1,1),\\
&I_i^c(2) = I_i(1,2),\\
&I_i^c(3) = I_i(2,1),\\
&I_i^c(4) = I_i(3,1),\\
&I_i^c(5) = I_i(2,2),\\
&\quad \cdots\cdots\\
&I_i^c(MN/(mn)) = I_i(M/m, N/n).
\end{aligned} \tag{16}$$

Thus, we obtain mn column vectors I_i^c, $i = 1, 2, \cdots, m \times n$. Then, these column vectors are combined into a new matrix $C = [I_1^c, I_2^c, \cdots, I_{mn}^c]$ with size $((MN)/(mn)) \times (mn)$. Fig. 3 shows a transposed matrix C from Lenna image, we can see that the column vector in C is very similar and highly correlated, which is natural to be decomposed using NMF, since NMF is originally designed to decompose column-similar matrices. Thus, we used NMF to decompose the matrix C, i.e., $C \approx WH$ and produce two nonnegative matrix W and H, where W is the normalized basis column vectors and H is the encoding coefficient vectors. Here, the matrix H involves the main local features under the basis matrix W, and these features are marked as larger encoding coefficients.

In the proposed scheme, the watermark M is embedded into the l maximum elements of the matrix H randomly. Suppose the watermark element w_k is embedded in the coef-

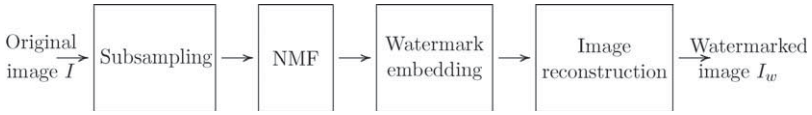


Fig. 2. Watermark embedding process.

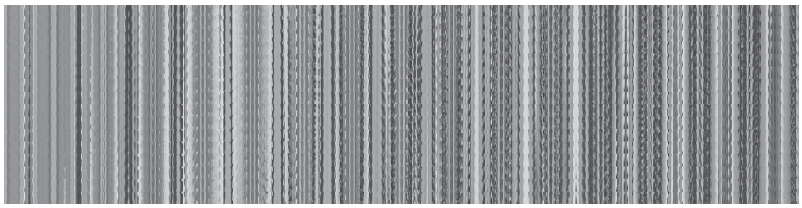


Fig. 3. The transposed matrix C constructed from the subimages, where the subsampling parameter $m, n = 16$.

ficient H_{ij}, the embedding algorithm is

$$H_{ij} \leftarrow H_{ij} + \alpha w_k, \tag{17}$$

where, α denotes the watermark embedding strength. Once all the watermark elements are embedded, the new matrix C are constructed using $C \leftarrow WH$, then through image reconstruction and inverse subsampling, the watermarked image I_w is obtained. In the watermark embedding process, due to good capability of local decomposition for non-negative matrix factorization, the maximum elements in the matrix H denotes the local most distinct features under the basis matrix W. Thus, the scheme can achieve a better local watermark embedding algorithm, and the robustness will be demonstrated in the following experiments.

4.2. *Watermark Detection*

Given a suspected watermarked image I^*, for the watermark detection, firstly it is decomposed using subsampling into subimages, the subimages are spread into column vectors as that in the watermark embedding process, and the vectors are combined into a matrix C^*, then NMF is applied to C^*, i.e., $C^* \approx W^* H^*$. Suppose the watermark w_k is suspected to embedded in the matrix element H^*_{ij}, we use the following similarity detection to make a decision on whether the image is watermarked as follows:

$$\rho = \sum_{k=1}^{l} w_k H^*_{ij}. \tag{18}$$

Then given a detection threshold T, if $\rho \geqslant T$, we think the image I^* is watermarked using M, otherwise not.

5. Experiments

In this section, some experiments are carried out to evaluate the performance of the proposed scheme. Firstly, we test the proposed scheme using the popular test image, Lenna, shown in Fig. 4(a). In the watermarking process, the subsampling parameters $m, n = 16$ and $l = 1000$, and the watermark embedding strength $\alpha = 0.03$. Fig. 4(b) shows the watermarked image, and its watermark detection value $\rho = 31.1$. We also used other 1000 random watermark seeds to test the detection process, and the histogram of the detection value is shown in Fig. 5, where only one is the correct watermark. If we set the detection threshold $T = 10$, only the correct watermark can be detected.

We tested the robustness of the proposed watermarking scheme using some signal processing distortions, and also we compared the performance of the proposed scheme with the classical DCT based spread spectrum watermarking scheme proposed in (Cox *et al.*, 1997). Fig. 6 shows some experimental results under JPEG compression, Gaussian lowpass filtering and noise addition with different PSNR, which are obtained through

(a) (b)

Fig. 4. (a) The original image, Lenna. (b) A watermarked version of (a).

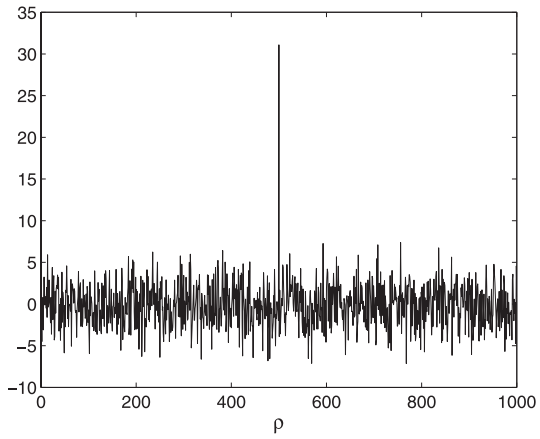
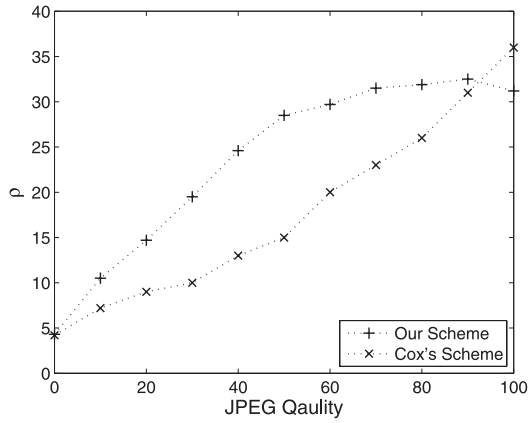


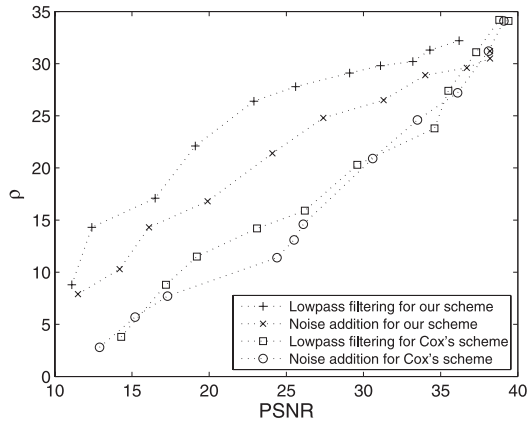
Fig. 5. The response of 1000 watermark seeds, where only one is the correct watermark.

setting different filter parameters and noise strength using Matlab. We can see that the proposed scheme achieves a strong robustness to common signal processing, furthermore, compared with Cox's classical scheme, it is also more robust. Furthermore, Stirmark is also used to test the scheme using some geometrical and other distortions. Table 1 shows some robustness test results under rotation, cropping and resizing, which also show better robustness than Cox's scheme. Especially, we can see that if the detection threshold T is set to 10, the watermark can still be detection under cropping with better robustness, while Cox's scheme fails.

In order to further evaluate the performance of the proposed scheme, we also build an image database including 1000 test images. Firstly, we used the database to test the robustness under JPEG compression. Fig. 7(a) shows the histogram of the detection value ρ under JPEG quality 60, which shows that if the threshold T is set to 10, all of the images can be thought to be watermarked. Then, we test the robustness under geometrical



(a)



(b)

Fig. 6. Robustness under signal processing. (a) JPEG compression. (b) Gaussian lowpass filtering and noising.

Table 1
Robustness experiments under geometric attacks for the proposed scheme and Cox's scheme

Attacks	ρ	
	Our Scheme	Cox's Scheme
cropping (10%)	21.5	8.4
cropping (20%)	18.8	9.1
cropping (30%)	16.9	6.3
Resizing (1.2)	27.7	21.9
Resizing (0.8)	28.8	20.2
Rotation ($-60°$)	33.3	28.1
Rotation ($40°$)	33.8	27.6

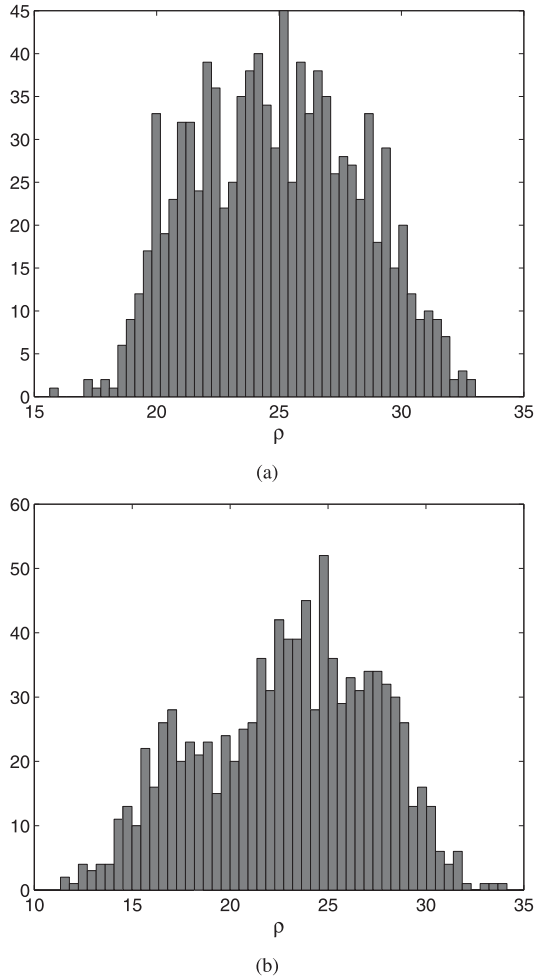


Fig. 7. The histogram of the detection value ρ. (a) 1000 test images under JPEG quality 60. (b) 1000 test images under Gaussian lowpass filtering.

cropping, the percentage for cropping is from 10% to 30%. Fig. 7(b) shows the histogram of the detection value under cropping, where, if the threshold is set to 10, over 98% of the images can be certificated for the corrected watermark, which also shows a good robustness. Furthermore, we also test many other performance with the database, including noising, lowpass filtering, rotation, resizing, etc. all of the experiments show that the proposed watermarking scheme is quite robust to theses attacks.

Finally, we also test the robustness under permutation attack. In (Lu *et al.*, 2005), we proposed a simple permutation attacks, which fail most of current subsampling based watermarking schemes through random permutating the corresponding elements in every subimages. Table 2 gives some robustness test results under common permutation attacks, which shows strong robustness to the permutation attack.

Table 2
Experimental results for common permutation attacks (Lu *et al.*, 2005)

Test image	Watermarked image		Attacked image	
	$PSNR$	sim	$PSNR$	sim
Lenna	37.1857	31.1	36.2732	30.2
Baboon	34.2104	33.2	32.4481	33.5
Barbara	35.7693	29.4	33.4378	28.6

6. Conclusions

In this paper, we have proposed a novel image watermarking based on subsampling and nonnegative matrix factorization. In the scheme, a pseudo-random Gaussian watermark sequence is embedded in the encoding vectors of NMF in subsampling domain. Experimental results shows that the proposed scheme achieve strong robustness to many signal processing and distortions due to the subimage construction for subsampling and physically meaningful factorization of NMF.

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W. Lu received his BSc degree in automation from Northeast University, China in 2002 and his MSc and PhD degrees in computer science from Shanghai Jiao Tong University, China in 2005 and 2007 respectively. He joined the Department of Computer Science, Sun Yat-sen University, China in 2007. His current research interests include multimedia forensics, digital watermarking, pattern recognition and image processing.

H. Lu received his PhD degree in electrical engineering from Southeast University, Nanjing, China, in 1997. He is currently a professor with the Department of Computer Science and Engineering, Shanghai Jiao Tong University, China. His research interests include neural networks, chaos and complex networks, image processing and pattern recognition.

Perdiskretizavimu ir neneigiamos matricos faktorizavimu pagrįstas patikimas vandens ženklų įterpimas

Wei LU, Hongtao LU

Šis straipsnis siūlo naują patikimą schemą, naudojančią perdiskretizavimą ir neneigiamos matricos faktorizavimą, vandens ženklams skaitmeniniame paveiksle įterpti. Pirmiausia atliekamas perdiskretizavimas paveikslo dalinei sekai gauti. Tada remiantis stulpelių panašumu, seka išskaidoma faktorizuojant neneigiamą matricą. Gausinė pseudo-atsitiktinė vandens ženklų seka yra įterpiama į faktorizuotus išskaidymo koeficientus. Dėl didelio dalinių paveikslų panašumo ir prasmingo faktorizavimo pasiūlyta schema yra patikima, ypač prieš dažnai naudojamą perstatymų ataką. Skaičiuojamasis eksperimentas demonstruoja pasiūlytos schemos gerumą.