On the Optimization of Investment Strategies in the Context of Virtual Financial Market by the Individual Approach to Risk

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Abstract. The optimal financial investment (Portfolio) problem was investigated by leading financial organizations and scientists. Nobel prizes were awarded for the Modern Portfolio Theory (MPT) and further developments. The aim of these works was to define the optimal diversification of the assets depending on the acceptable risk level.

In contrast, the objective of this work is to provide a flexible, easily adaptable model of virtual financial markets designed for the needs of individual users in the context of utility theory. The aim is to optimize investment strategies. This aim is the new element of the proposed model and simulation system since optimization is performed in the space of investment strategies; both short term and longer term.

The new and unexpected result of experiments with the historical financial time series using the PORTFOLIO model is the observation that the minimal prediction errors do not provide the maximal profits.

Key words: optimization, portfolio problem, utility theory, Nash equilibrium, stock exchange.

1. Introduction

The problem of optimal investment was discussed in many well-known publications. Recently, the practical importance of the problem has been increasing when major investors are accompanied by millions of small stockholders which decide how to use their savings better without financial education and experience.

The traditional approach is represented by the Modern Portfolio Theory (MPT). MPT authors Markowitz (1959, 1952), Merton (1972) and Sharpe (1994) who developed methods of reflecting the investment risk (Sharpe Ratio) were awarded the Nobel prizes.

The recent developments and applications of MPT are discussed in A number of investment organizations are making decisions using the software based on the theoretical results of Black F, Scholes (1972). Some limitations of these theories and their applications have been noticed during the recent financial crisis when the investors experienced considerable losses (Krugman, 2009).

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In Brunnermeier and Pedersen (2007), the authors provide a model that links an assets market liquidity, i.e., the ease with which it is traded, and traders' funding liquidity, i.e., the ease with which they can obtain funding. The model explains the empirically documented features that the market liquidity can suddenly dry up.

The paper Brunnermeier and Julliard (2006) describes a formal model explaining how a reduction in inflation can fuel run-ups in housing prices, if people suffer from money illusion. The paper of Brunnermeier (2007) studies portfolio holdings and asset prices. The paper of Brunnermeier and Yogo (2009) shows by examples that, when a company is unable to rollover its debt, it may have to seek more expensive sources of financing or even liquidate its assets. In Brunnermeier and Nagel (2006), data from the Panel Study of Income Dynamics (PSID) was used to investigate how households' portfolio allocations change in response to wealth fluctuations.

Financial market simulators are developed to satisfy the needs of small individual investors. The examples are the "StockTrak global portfolio simulator", the "MarketWatch, virtual stock exchange", and the "Stock Simulator" of Investopedia. Some banks offer their own investment simulators such as the "Barclays Fantasy Investment Game". Users of these simulators working with "Virtual Stocks" are informed about the results. The graphical user interfaces are friendly. However, the theoretical basis of these models and computing algorithms remains unknown. So the users cannot grasp the reasons why they win and why they experience losses.

To present the individual stockholders with a tool where everything is open is one of the aims of the PORTFOLIO model introduced in this paper. To accomplish this task, we deviate from the traditional portfolio models since this model is for defining the best investment strategies but not just the best diversification of assets. Thus, PORTFOLIO is just a conditional name for short. The second aim is to apply the utility theory (Fishburn, 1964) in the risk assessment and to investigate the relation of virtual and real financial markets to the Nash equilibrium (Nash, 1951). These are new elements of this work.

In contrast to well-known results, this work not only simulates traditional results of utility and portfolio theories, but also complements them by various investment procedures and renders a possibility to users to develop and implement their own investment strategies.

This is important for individual users with different approaches to risk. Thus, the model can be used in financial studies and scientific collaboration. For example, the model can be applied in the experimental investigation of the Nash equilibrium by simulating both virtual and real financial markets at the same time. This can provide some additional data for discussions about the rationality of investors behavior. It is important for theory. For example, the Nobel prize winner John Nash assumes rational behavior (Nash, 1951) where another Nobel winner Krugman (2000, 2008, 2009) is critical of this assumption.

Thus, the model can be useful to studies, to scientific collaboration, and to stockholders who are solving optimal investment problems and regard risk in an individual way. This can be explained by some new elements of the paper such as application of individual utility functions that define the personal preferences and investigation of Nash equilibrium by simulating and comparing different investment strategies in virtual and real financial markets.

Risk evaluation by individual utility functions is basically different from the traditional evaluation by the return variance. The reason is that individual investors are risk-prone regarding small sums and risk averse while investing large sums thus. In such cases we need to solve the problems of global optimization, since the utility functions are not convex. This is not considered in MPT ant its developments. In this work. the corresponding global optimization procedures are developed and implemented. In addition, the algorithms and software for testing the personal utility functions are created. These new tools are intended to help individual stockholders to select investment strategies according to their personal preferences.

The market prediction and portfolio optimization were regarded in most of the financial market research. The investigation of different investment strategies including those which are close to the Nash equilibrium is the feature of this work.

The investment problems were investigated in many publications. Let us mention some examples. In Raudys and Raudys (2011, 2012), decisions of portfolio management are considered in the context of artificial intelligence. In the paper of Ramanauskas and Rutkauskas (2009), an artificial stock market is simulated by learning agents. The visual analysis of data sets (Medvedev *et al.*, 2011) could be applied, too.

In Mockus (2012), a preliminary investigation of the virtual stock exchange of a single stock is discussed. The results of these papers helped to initiate this work that simulates the optimal investment problem in the multi-stock market. Therefore we just refer to paper of Mockus (2012) for a mathematical description of the singe stock model and describe only the specific expressions of multi-stock models. However, some definitions and expressions that describe the buying–selling strategies and investors' profit are repeated for convenience of reading.

2. The PORTFOLIO Model of a Virtual Financial Market

2.1. *Buying and Selling Strategies*

We consider a virtual market of I major players $i = 1, \ldots, I$ and $j = 1, \ldots, J$ stocks. The following notation is used:

$z(t, j) = z(t, i, j)$ is the price of stock j at time t, predicted by the player i,

$Z(t, j)$ is the actual[2] price at time t,

$U(t, j) = U(t, i, j)$ is the actual profit accumulated at time t by the player i buying–selling stock j,

$\delta(t, j)$ is the dividend of stock j at time t,

$\alpha(t)$ is the yield at time t,

$\gamma(t)$ is the bank interest at time t,

$\beta(t, i) = \beta(t, i, j)$ is the relative stock j price change at time t as predicted by the player i

$$\beta(t, i, j) = \big(z(t + 1, i, j) - Z(t, j)\big)\big/Z(t, j). \tag{1}$$

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[2]The term 'actual' means simulated.

Expected profitability[3] (relative profit) $p(t, i, j)$ of an investment at time t depends on the predicted change of stock prices $\beta_i(t, j)$, dividends $\delta_i(t, j)$, the yield $\alpha(t)$, and the interest $\gamma(t)$

$$p(t, i) = \begin{cases} \beta(t) + \delta(t) - \gamma(t), & \text{investing borrowed money,} \\ \beta(t) + \delta(t) - \alpha(t), & \text{investing own money.} \end{cases} \tag{2}$$

The aim is profit, thus a customer i will buy some amount $n_b(t, i, j) \geqslant n(t, j)$ of stocks j, if profitability is greater comparing with the relative transaction cost $\tau(t, n)$; $p(t, i, j) > \tau(t, n)$, and will sell stocks, if the relative loss (negative profitability $-p(t, i, j)$) is greater as compared with the transaction cost $p(t, i, j) < -\tau(t, n)$, or will do nothing, if $-\tau(t, n) \leqslant p(t, i, j) \leqslant \tau(t, n)$. Here the relative transaction cost is defined as the relation

$$\tau(t, n) = \frac{\tau_0}{n(t, j)Z(t, j)}, \tag{3}$$

where τ_0 is the actual transaction cost and $n = n(t, j)$ is the number of transaction stocks. It follows from the equality $\tau(t, n) = p(t, i, j)$ that the minimal number of stocks to cover transaction expenses is

$$n(t, j) = \frac{\tau_0}{p(t, i, j)Z(t, j)}. \tag{4}$$

Therefore, the buying–selling strategy $S(t, i, j)$ of stock j by the customer i at time t in terms of profitability levels is as follows

$$S(t, i, j) = \begin{cases} \text{buy } n_b(t, i) \geqslant n(t, j) \text{ stocks,} & \text{if } p(t, i, j) \geqslant \tau(t, n) \text{ and } n \leqslant n_b^{\max}, \\ \text{sell } n_s(t, i, j) \geqslant n(t, j) \text{ stocks,} & \text{if } p(t, i, j) \leqslant -\tau(t, n) \text{ and } n \leqslant n_s^{\max}, \\ \text{wait,} & \text{if } |p(t, i, j)| \leqslant \tau(t, n^{\max}). \end{cases} \tag{5}$$

Here $n^{\max} = \max(n_b^{\max}, n_s^{\max})$, where $n_b^{\max}$ is the maximal number of stocks to buy, and $n_s^{\max}$ is the maximal number of stocks to sell. If

$$n_b(t, i, j) = n_b^{\max} \quad \text{and} \quad n_s(t, i, j) = n_s^{\max}, \tag{6}$$

then this buying–selling strategy reflects the behavior of risk-neutral stockholders which invest all available resources if the expected profitability is higher than the transaction cost. If the expected losses are greater, then all the stocks are sold. It means that stockholders may tolerate a considerable probability of losses, if the expected profits are positive. In this way, the maximal expected profit is provided. However, the probability to get losses instead of profits could be near to 0.5.

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[3]The term "profit" can define losses if negative terms prevail.

From expressions (1) and (2), the buying–selling strategy $S(t, i, j)$ in terms of stock price levels is

$$S(t, i, j) = \begin{cases} \text{buy } n_b(t, i, j) \geqslant n(t, j) \text{ stocks,} \\ \quad \text{if } Z(t, j) \leqslant z_b(t, n, i, j) \text{ and } n \leqslant n_b^{\max}, \\ \text{sell } n_s(t, i, j) \geqslant n(t, j) \text{ stocks,} \\ \quad \text{if } Z(t, j) \geqslant z_s(t, n, i, j) \text{ and } n \leqslant n_s^{\max}, \\ \text{wait,} \quad \text{otherwise.} \end{cases} \tag{7}$$

Here the price level of the player i to buy at least $n = n(t, j)$ stocks at time t is

$$z_b(t, n, i, j) = z(t + 1, i, j) \big/ \big(1 - \delta(t) + \alpha(t) + h(t) + \tau(t, n)\big). \tag{8}$$

The price level of the player i to sell at least $n = n(t, j)$ stocks at time t is

$$z_s(t, n, i, j) = z(t + 1, i, j) \big/ \big(1 - \delta(t) + \alpha(t) + h(t) - \tau(t, n)\big), \tag{9}$$

where $z(t + 1, i, j)$ is the stock j price predicted by the investor i at time $t + 1$.

The market buying price at time t is the largest buying price of players $i = 1, \ldots, I$, $z_b(t, n) = z_b(t, n, i, j^{\max})$, where $i^{\max} = \arg\max_i z_b(t, n, i, j)$.

The market selling price at time t is the lowest selling price of players $i = 1, \ldots, I$, $z_s(t, n) = z_b(t, n, i, j^{\min})$, $i^{\min} = \arg\min_i z_s(t, n, i, j)$.

2.2. *Price Simulation*

2.2.1. *Buying–Selling Price*

The market buying price of stock j at time t is the largest buying price of players $i = 1, \ldots, I$, $z_b(t, n, j) = z_b(t, n, i, j^{\max})$, where $i^{\max} = \arg\max_i z_b(t, n, i, j)$.

The market selling price at time t is the lowest selling price of players $i = 1, \ldots, I$, $z_s(t, n, j) = z_s(t, n, i, j^{\min})$, $i^{\min} = \arg\min_i z_s(t, n, i, j)$.

The number of stocks j owned by the player i at time $t + 1$ is

$$N(t + 1, i, j) = \begin{cases} N(t, i, j) + n_b(t, n, i, j), & \text{if } Z(t, j) < z_b(t, n, j), \\ N(t, i, j) - n_s(t, n, i, j), & \text{if } Z(t, j) > z_s(t, n, j), \\ N(t, i, j), & \text{if no deal.} \end{cases} \tag{10}$$

Here $n_b(t, n, i, j)$ and $n_s(t, i, j)$ are the numbers of stocks j for buying and selling operations by the player i at time t.

2.3. *Investors Profit*

The product $N(0, i, j)\, Z(0, j)$ is the initial investment to buy $N(0, i, j)$ shares j using an investors' own capital at the initial price $Z(0, j)$. The initial funds to invest are $C_0(0, i)$ and the initial credit limit is $L(0, i)$.

$L(t, i)$, $t = 1, \ldots, T$ is the credit available for a customer i at time t. The investors' own funds in cash $C_0(t, i)$ available for investing at time t are defined by the recurrent expression

$$C_0(t, i) = C_0(t - 1, i) - \sum_j \big(N(t, i, j) - N(t - 1, i, j)\big)Z(t, j), \tag{11}$$

where $t = 1, \ldots, T$. Here the product $(N(t, i, j) - N(t - 1, i, j))Z(t, j)$ defines the money involved in buying–selling stocks.

Stocks are obtained using both investor's own money $C_0(t, i)$ and the funds $b(t, i)$ borrowed at the moment t. The borrowed sum of the stockholder i accumulated at time t is

$$B(t, i) = \sum_{s=1}^{t} b(s, i), \tag{12}$$

the symbol $b(t, i)$ shows what the user i borrows at the moment t.

The general borrowing expenses are

$$B_{sum}(t, i) = B(t, i) + \sum_{s=1}^{t} B(s, i)\gamma(s, i), \tag{13}$$

where the first term denotes the loan accumulated at time t and the second term shows the interest.

An investor i gets a profit as the difference between the income from selling and buying stocks $D(t, i)$ and expenses for the borrowing funds $B_{sum}(t, i)$

$$U(t, i) = C_0(t, i) + D(t, i) - B_{sum}(t, i), \tag{14}$$

where

$$D(t, i) = \sum_j N(t, i, j)Z(t, j) - N(0, i, j)Z(0, j). \tag{15}$$

The investors i profit at the end of investment period is denoted by

$$U_i = U(T, i). \tag{16}$$

The funds available for the investor i at time t are

$$C(t, i) = C_0(t, i) + L(t, i) - B_{sum}(t, i). \tag{17}$$

An investor is trying to maximize gains by borrowing money to invest in shares that appreciate more than what it costs him by way of interest. It means leveraging shares for an investment.

The number $n_b(t.j)$ of stocks j to buy at the time t is restricted by the following inequalities

$$\sum_j n(t, j)Z(t, j) \leqslant C(t, i) \tag{18}$$

and

$$n(t, j) \leqslant n_b(t, i, j), \tag{19}$$

where the last inequality restricts the transactions costs. The stockholder will be insolvent at the time $t = t_i^*$ if the loan exceeds the assets

$$B_{sum}(t_i, j) > C_0(t_i, j) + L(t_i, j) + \sum_j N(t, i, j)Z(t, j), \tag{20}$$

since there will not be enough money to pay back all the borrowing expenses $B_{sum}(t_i^*, i)$. This can happen without buying additional stocks, because the interest $B_{sum}(t, i)$ accumulates automatically.

Part of Profit by Stock j. In longer term investment strategies using the Sharpe ratio, the general profit should be divided between different stocks. Denote by $C_0(0, i, j) \leqslant L(0, i, j)$ the initial funds to be invested in the stock $j = 1, \ldots, J$, where $L(0, i, j)$ is the initial credit limit for stock j and

$$\sum_j L(0, i, j) = L(0, i). \tag{21}$$

For example, the initial funds may be divided into equal parts

$$\sum_j C_0(0, i, j) = C_0(0, i)/J. \tag{22}$$

The investors' own funds in cash $C_0(t, i, j)$, accumulated buying–selling stocks j and available for investing at time t in the stock j, are defined by the recurrent expression below

$$C_0(t, i, j) = C_0(t - 1, i, j) - \big(N(t, i, j) - N(t - 1, i, j)\big)Z(t, j), \tag{23}$$

where $t = 1, \ldots, T$. Here the product $(N(t, i, j) - N(t - 1, i, j))Z(t, j)$ defines the money involved in buying–selling stocks j.

Stocks are obtained using both investors' own money $C_0(t, i, j)$ and the funds $b(t, i, j)$ borrowed at the moment t. The borrowed sum of the stockholder i for the stock j accumulated at time t is

$$B(t, i, j) = \sum_{s=1}^{t} b(s, i, j), \tag{24}$$

where

$$\sum_j B(t, i, j) = B(t, i). \tag{25}$$

The symbol $b(t, i, j)$ shows what the user i borrows at the moment t for the stock j.

The general borrowing expenses for stock j are

$$B_{sum}(t, i, j) = B(t, i, j) + \sum_{s=1}^{t} B(s, i, j)\gamma(s, i), \tag{26}$$

where the first term denotes the loan accumulated at time t, the second term shows the interest, and

$$\sum_j B_{sum}(t, i, j) = B_{sum}(t, i). \tag{27}$$

The investor i gets a profit as the difference between the income from selling and buying stocks $D(t, i, j)$ and expenses for the borrowing funds $B_{sum}(t, i, j)$

$$U(t, i, j) = C_0(t, i, j) + D(t, i, j) - B_{sum}(t, i, j), \tag{28}$$

where

$$D(t, i, j) = N(t, i, j)Z(t, j) - N(0, i, j)Z(0, j). \tag{29}$$

The investor's i profit from the stock j at the end of investment period is denoted as

$$U_{i.j} = U(T, i.j), \tag{30}$$

where

$$\sum_j U_{i.j} = U_i. \tag{31}$$

If for some reason equalities (25), (27), and (31) are violated, then the normalization of components may be applied to restore them.

The bank profit expressions are the same as in the single stock market model of Mockus (2012).

2.4. *Multi-Level Operations*

In the opinion of some professional brokers we have interviewed, one needs at least three buying profitability levels $p_b(t, i, j, l), l = 1, 2, 3$, where

$$p_b(t, i, j, l+1) > p_b(t, i, j, l), \qquad p_b(t, i, j, 1) = \tau(t), \tag{32}$$

and three selling profitability levels $p_s(t, i, j, l), l = 1, 2, 3$, where

$$p_s(t, i, j, l+1) < p_s(t, i, j, l), \qquad p_s(t, i, j, 1) = -\tau(t),$$
$$p_b(t, i, j, l) > p_s(t, i, j, 1). \tag{33}$$

to explain the behavior of major stockholders. The level $l = 1$ means to buy-sell the minimal number of stocks. The level $l = 3$ means to buy-sell as much stocks as possible, and the level $l = 2$ is in the middle. Details of multi-level operations are in Mockus (2012).

3. Price Prediction

In this model, two versions of AutoRegressive models (AR(p) and AR-ABS(p)) and a version of AutoRegressive Moving Average (ARMA-ABS(p, q)) model are considered for stock rate predictions. AR(p) uses traditional least squared approach. AR-ABS(p) and ARMA-ABS(p, q) minimize the absolute errors.

The development and implementation of ARMA-ABS(p, q) is a new feature of this work. The common disadvantage of both the new ARMA-ABS(p, q) and traditional ARMA(p, q) models is that estimation of moving average parameters q needs global optimization and the exact value of global minimum remains unknown, only some approximation is obtained after lengthy calculations.

Thu, we use ARMA-ABS(p, q) just for comparison with the simple AR-ABs(p) model which can be efficiently used for daily predictions directly in the PORTFOLIO model. No significant improvement of prediction accuracy was noticed. The AR-ABS model was described in Mockus (2012). The description of ARMA-ABS follows.

3.1. *ARMA-ABS(p, q) Model*

3.1.1. *Definition of Residuals*
Denote by $Z(s)$ the stock price at time $s \leqslant t$. Denote by $a = (a_1, \ldots, a_p)$ a vector of auto-regression (AR) parameters, and by $b = (b_1, \ldots, b_q)$ a vector of moving-average (MA) parameters. When describing these models, we omit the user index i, for simplicity. Then the residual

$$\epsilon_s = Z(s) - \sum_{k=1}^{p} a_k Z(s-k) + \sum_{j=1}^{q} b_j \epsilon_{s-j}. \tag{34}$$

3.1.2. *Optimization of AR Parameters*
The optimal prediction parameters $a = a(b)$ as a function of b are defined by the following condition

$$a_k = \arg \min_{a_k} \sum_{s=1}^{t} |\epsilon_s|. \tag{35}$$

To solve (35) we apply linear programming.

3.1.3. *AR-ABS(p) Model Using Auxiliary Variables* u, v, ϵ

$$\min_{u,v,\epsilon} \sum_{s=1}^{t} u_s, \tag{36}$$

$$u_s \geqslant Z(s) - \sum_{k=1}^{p} v_k Z(s-k) + \sum_{j=1}^{q} b_j \epsilon_s, \quad s = 1, \ldots, t, \tag{37}$$

$$u_s \geqslant -Z(s) + \sum_{k=1}^{p} v_k Z(s-k) - \sum_{j=1}^{q} b_j \epsilon_s, \quad s = 1, \ldots, t, \tag{38}$$

$$u_s = u_{s1} - u_{s2}, \quad s = 1, \ldots, t, \tag{39}$$

$$\epsilon_s = Z(s) - \sum_{k=1}^{p} v_k Z(s-k), \tag{40}$$

$$\epsilon_s = \epsilon_{s1} - \epsilon_{s2}, \quad s = 1, \ldots, t, \tag{41}$$

$$v_k = v_{k1} - v_{k2}, \quad k = 1, \ldots, p, \tag{42}$$

where $u_{s1} \geqslant 0$, $u_{s2} \geqslant 0$, $v_{k1} \geqslant 0$, $v_{k2} \geqslant 0$, $y_j = y_{j1} \geqslant 0$, $y_{j2} \geqslant 0$, $\epsilon_{s1} \geqslant 0$, $\epsilon_{s2} \geqslant 0$ are the auxiliary variables. The scale parameters $a_k.b_j$ are obtained by solving the LP problem as the corresponding auxiliary variables $a_k^i = v_k$, $b_j = y_j$.

3.1.4. *Optimization of MA Parameters*

The sum of absolute residuals is a multimodal function of parameters b with the number of extrema depending on the length of the time series. Thus, we have to use the global optimization algorithms. Figure 1 illustrates how the Mean Absolute Error (MAE) depends on ARMA-ABS(p, q) parameters $(b_1, b_2) \in [0.5, 05]$ using historical Microsoft data.

In particular, the figure shows that MAE, as a function of two parameters has several local minima. In general, the number of local minima depends on the length of observer. Denote

$$f(b) = \log S(a(b), b), \tag{43}$$

where

$$S(a(b), b) = \sum_{s=1}^{t} |\epsilon_s|, \tag{44}$$

$a = (a_1, \ldots, a_p)$ and $b = (b_1, \ldots, b_q)$.

Here $a(b)$ is in (35) at the fixed parameter b. Denote
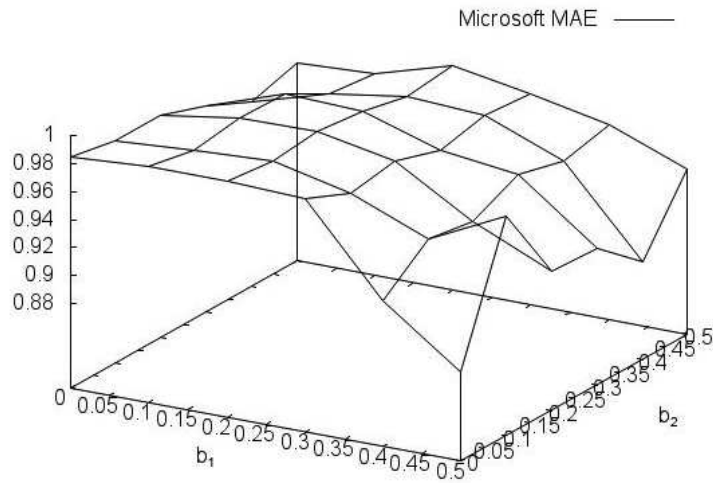
$$b^0 = \arg\min_b f(b). \tag{45}$$

Fig. 1. MAE as a function of parameters b_1 and b_2 by historical Microsoft data.

The simplest global optimization algorithm is Monte Carlo. A number of global optimization methods are implemented as web-based Java applets (Mockus, 2006).

4. A Set of Investment Strategies

In this section, we first consider four heuristic investment strategies representing personal opinions of some real stockholders with different approaches to risk. An advantage is the simplicity of these procedures that allow daily updates. It is important in the short-term investment.

Considering longer term investment, two additional strategies are described. The first one estimates the risk using bankruptcy probabilities. The second longer term strategy maximizes the Sharpe ratio. An advantage of these strategies is some theoretical base. A disadvantage is the long computing time. Therefore, the strategies are suitable for longer term investment. In the longer term experiments, the data set is divided into the learning and testing sets where the efficiency of investment strategies is tested.

4.1. *Short Term Investments*

Consider operations that involve different stocks denoted by indexes $j = 1, \ldots, J$. Denote by $p(t, i, j)$ the profitability of the jth stock for a customer i at time t. Denote by $j^{\max}$ the stock with the highest profitability:

$$j^{\max} = \arg \max_{j} p(t, i, j), \tag{46}$$

4.1.1. *Strategy No. 1: Risk-Averse Stockholders: Buying the Best – Selling the Losers by Three Profitability Levels*

First, the stockholder i sells all nonprofitable stocks

$$p_s(t, i, j) \leqslant -\tau(t, i, j), \tag{47}$$

and then invests all the available funds to buy the most profitable stock. The stockholder i does not sell the stock j, if the expected loss is smaller than the transaction cost $|p(t, i, j)| < \tau(t, i, j)$. We assume that transaction costs τ are the same for all stocks and do not depend on time. However, extending expression (3) of relative transaction costs to multi-stock case we use indexes (t, i, j) instead of (t, n), since these costs depend on the numbers n of stocks j involved in the operation at time t by the stockholder i.

This selling strategy reflects risk-aware users which keep some less profitable stocks to avoid possible losses if predictions happen to be wrong.

Note that the risk-neutral users sell all the stocks with profitability lower then maximal and then invest all the available funds in the stock $j^{\max}$ which provides the maximal return. This way they maximize the expected profit. Details are in the next section.

The investors' i own funds at time t, including the income from selling unprofitable stocks, are expressed as the sum

$$C_0(t, i) = \sum_j C_0(t, i, j), \tag{48}$$

where $C_0(t, i, j)$ is defined by the following recurrent expression

$$C_0(t, i, j) = C_0(t - 1, i, j) - \big(N(t, i, j) - N(t - 1, i, j)\big)Z(t, j). \tag{49}$$

The investors' funds available for investing are

$$C(t, i) = C_0(t, i, j) + L(t, i) - B_{sum}(t, i), \tag{50}$$

here $t = 1, \ldots, T$, $L(t, i)$ is the credit limit at time t, and $B_{sum}(t, i)$ is the borrowed sum.

Then we invest all available resources to buy the most profitable stock $j^{\max}$. It means that we sell stocks as the risk aware user but we buy stocks as the risk-neutral one. Thus, the feasible number of stocks $j = j^{\max}$ to buy at time t is as follows

$$n_b\big(t, i, j^{\max}\big) = int\big(C(t, i)\big/Z\big(t, j^{\max}\big)\big), \quad \text{if } p\big(t, i, j^{\max}\big) > \tau\big(t, i, j^{\max}\big). \tag{51}$$

4.1.2. *Strategy No. 2: Risk-Aware Stockholders: Selling all Unprofitable Stocks – Buying the Best Ones*

First, the stockholder i sells all the nonprofitable stocks

$$p_s(t, i, j) \leqslant -\tau(t, i, j) \tag{52}$$

and then invests all the available funds to buy the most profitable stock. The stockholder i does not sell the stock j, if the expected loss is smaller than the transaction cost $|p(t, i, j)| < \tau(t, i, j)$. We assume that transaction costs τ are the same for all stocks and do not depend on time. However, extending expression (3) of relative transaction costs to the multi-stock case we use indexes (t, i, j) instead of (t, n), since these costs depend on the numbers n of stocks j involved in the operation at time t by the stockholder i.

This selling strategy reflects risk-aware users which keep some less profitable stocks to avoid possible losses if predictions happen to be wrong.

Note that the risk-neutral users sell all the stocks with a profitability smaller than maximal and then invest all the available funds in the stock $j^{\max}$ which provides the maximal return. This way they maximize the expected profit. Details are given in the next section.

The investors' i own funds at time t, including the income from selling unprofitable stocks, are expressed as the sum

$$C_0(t, i) = \sum_j C_0(t, i, j), \tag{53}$$

where $C_0(t, i, j)$ is defined by this recurrent expression

$$C_0(t, i, j) = C_0(t - 1, i, j) - \big(N(t, i, j) - N(t - 1, i, j)\big)Z(t, j). \tag{54}$$

The investors' funds available for investing are

$$C(t, i) = C_0(t, i, j) + L(t, i) - B_{sum}(t, i), \tag{55}$$

here $t = 1, \ldots, T$, $L(t, i)$ is the credit limit at time t, and $B_{sum}(t, i)$ is the borrowed sum.

Thus, the feasible number of stocks $j = j^{\max}$ to buy at time t is as follows

$$n_b\big(t, i, j^{\max}\big) = int\big(C(t, i)\big/Z\big(t, j^{\max}\big)\big), \quad \text{if } p\big(t, i, j^{\max}\big) > \tau\big(t, i, j^{\max}\big). \tag{56}$$

4.1.3. *Strategy No. 3: Risk-Neutral Stockholders: Buying the Best Stocks and Selling all the Rest*

The risk-neutral stockholders use all available resources to buy the stock $j^{\max}$ which provides the highest expected profit:

$$j^{\max} = \arg\max_j p(t, i, j). \tag{57}$$

Denote by $J(\tau)$ a subset of stocks with profitability lower or equal to the best minus the relative transaction cost:

$$J(\tau) = \big\{j : \ p(t.i.j) \leqslant p\big(t, i, j^{\max}\big) - \tau\big(t, n_s(t, i, j)\big)\big\}, \tag{58}$$

where $n_s(t, i, j)$ is the number of stocks j for sale at time t by the stockholder i. Here, to define the relative transaction cost, we use a longer symbol $\tau(t, n_s(t, i, j))$ instead of the shorter one $\tau(t, i, j)$ to show the number of stocks $n_s(t, i, j)$ directly.

First, the risk-neutral stockholder is selling the stocks $j \in J(\tau)$ to raise funds for buying the single most profitable stock $j^{\max}$.

Stockholders do nothing, if the maximal expected profit is smaller than the transaction cost $C(t,i) p(t,i,j^{\max}) < \tau(t, n_b(t, j^{\max}))$ and they do not sell if the maximal expected losses are lower than $C(t,i) p(t,i,j^{\min}) < \tau(t, n_s(t, j^{\min}))$. Thus, the number of stocks $j = j^{\max}$ to buy at time t is as follows

$$n_b(t, j^{\max}) = int(C(t,i)/Z(t, j^{\max})), \quad \text{if } p(t,i,j^{\max}) \geqslant \tau(t, n_b(t, j^{\max})). \quad (59)$$

They also do not sell, if the maximal expected losses are smaller than the transaction cost $N(t,i,j^{\min}) Z(t,i) p(t,i,j^{\min}) < -\tau(t, n_s(t, j^{\min}))$, where $N(t,i,j)$ is the number of stocks j available at time t. The feasible number of stocks j to sell at time t

$$n_s(t, i, j^{\min}) = N(t,i,j), \quad \text{if } p(t,i,j) \leqslant p_s(t,i,j). \quad (60)$$

4.1.4. *Strategy No. 4: Risk-Averse Stockholders: Selling and Buying in Proportion to Profitability*

Denote by J_+ a set of stocks with a positive profitability, and by J_- the stocks with a negative profitability. Denote $J_b = |J_+|$ and $J_s = |J_-|$.

$$j_+^{\max} = \arg \max_{j \in J_+} p(t,i,j), \quad (61)$$

and

$$j_-^{\min} = \arg \min_{j \in J_-} p(t,i,j). \quad (62)$$

First, we sell stocks in proportion to $l = 1, \ldots, j_-^{\min}$ selling profitability levels $p_s(t,i,l) = p(t,i,j=l), l = 1, \ldots, j_-^{\min}$. Then we use all accumulated resources to buy stocks in proportion to $l = 1, \ldots, j_+^{\max}$ profitability levels $p_b(t,i,l) = p(t,i,j=l), l = 1, \ldots, j_-^{\max}$.

4.2. *Longer Term Investment*

4.2.1. *Estimates of Portfolio Profits in Real Market*

In the previous sections, we analyzed short term investing by daily decisions, for example. using different investment strategies. So, the search was in the strategy space. In this section, the best proportion of assets is defined using different approaches.

The traditional portfolio problem considers the optimal longer term diversity by defining optimal sharing of available resources among different assets (Markowitz, 1959, 1952; Merton, 1972).

The utility function approach is regarded in Strategy No. 5. In Strategy No. 6, we consider the maximization of the Sharp Ratio (Sharpe, 1994).

In the longer term investing models, the time series are split into learning and testing sets. In the learning stage, the mean and variance of portfolio $P(x)$ profit are estimated

using the first part of observations $1 \leqslant t_0 < T$, where $x = (x_j, \ j = 1, \ldots, J)$. Usually t_0 is about $T/2$. and the initial funds are equally divided among the stocks, meaning that $x_j^0 = C_0(0)/J$. Here $C_0(0)$ denotes initial funds of a single user.

Note that in virtual markets, the stock prices are generated by the interaction of different virtual investors.

In Strategy No. 5, the search for the optimal distribution of funds is performed by maximization of the utility function. In Strategy No. 6, the Sharpe Ratio is maximized. During the testing stage the profits of optimized portfolios are calculated using the remaining observations s: $t_0 < s \leqslant T$.

The data of the learning stage are used to estimate average deviations and variances. The sample mean of portfolio $P(x)$ that contains stocks with weights x_j, $j = 1, \ldots, J$ is as follows

$$m^u(x) = \sum_j x_j / x_j^0 m^{u_j}, \tag{63}$$

where

$$m^{u_j} = \frac{1}{t_0} \sum_{t=1}^{t_0} U(t, j). \tag{64}$$

Here $U(t, j)$ follows from (28) by omitting the investor's index i. The estimator of variance of the portfolio $P(x)$ is

$$\left(s^u(x)\right)^2 = \frac{1}{t_0 - 1} \sum_j \sum_k \sum_{t=1}^{t_0} x_j / x_j^0 \left(U(t, j) - m^{u_j}\right) x_k / x_k^0 \left(U(t, k) - m^{u_k}\right). \tag{65}$$

4.2.2. *Strategy No. 5: Definition of Risk by Survival Probabilities*

An important part of optimal investment is a definition of individual utility functions ϕ that determine particular investor's profit-to-risk relation (Fishburn, 1964). Now we consider an example how to invest some fixed capital.

The problem is to maximize the average utility of wealth. It is obtained by the optimal distribution of available capital among different assets with uncertain market prices. Denote by x_j the part of the capital invested into the object i. The returned wealth is $u_i = c_i x_j$. Here $c_i = 1 + \alpha_i$ and $\alpha_i > 0$ is the interest rate. Denote by $p_i = 1 - q_i$ the reliability of investment. Here q_i is the insolvency probability. $\phi(u)$ is the utility of the wealth y. Denote by $\Phi(x)$ the expected utility function. $\Phi(x)$ depends on the capital distribution $x = (x_1, \ldots, x_n)$, $\sum_i = 1$, $x_i \geqslant 0$ and the individual utility function $\phi(u)$. If the wealth is discrete $u = u^k$, $k = 1, \ldots, M$, the expected utility function

$$\Phi(x) = \sum_{k=1}^M \phi\left(u^k\right) p\left(u^k\right), \tag{66}$$

where M is the number of discrete values of wealth y^k. $p_x(u^k)$ is the probability that the wealth y^k will be returned, if the capital distribution is x. We search for such a capital distribution x that provides the greatest expected utility of the returned wealth:

$$\max_x \Phi(x), \tag{67}$$

$$\sum_{j=1}^{J} x_j = 1, \tag{68}$$

$$x_j \geqslant 0. \tag{69}$$

Real utility functions $\phi(u)$ are not convex, as usual, since investors tend to behave as risk-prone if small sums are involved. They are risk-averse when the funds are large. Thus, in order to maximize the utility functions $\Phi(x)$, defined by expressions (69) and (66), the global optimization methods should be applied.

Investing in stocks, apart from reliability p_i, $i = n + j$, $j = 1, \dots, m$ of companies, their future stock rates are uncertain, too. Denote the relative returned wealth of stock i as a_i. Then

$$a_{,i} = \begin{cases} \beta_i + \delta - \gamma & \text{investing borrowed money,} \\ \beta_i + \delta - \alpha & \text{investing own money,} \end{cases} \tag{70}$$

where β_i is the predicted change of stock prices , δ are dividends, α is the yield, and γ is the bank interest

Suppose that one predicts L different values of relative stock rates a_i^l, $l = 1, \dots, L$ with the corresponding estimated probabilities p_i^l, $\sum_{l=1}^{L} p_i^l = 1$, $p_i^l \geqslant 0$.

Assuming that bankruptcy probabilities p_i of different stocks i are independent, one may define the probabilities $p(u^i)$ of different discrete values of the wealth y^i, $i = 1, \dots, n + m$ by exact expressions.

This approach includes not only the profit prediction, but also the prediction of bankruptcy probabilities. The bankruptcy probabilities are considered in Hillegeist *et al*. (2004). For the profit predictions we use simplest autoregressive models $AR(p)$ and AR-ABS(p). The ARMA-ABS(p, q) model was used just for testing autoregressive moving average models, since it demands to much of computing time.

4.2.3. *Strategy No. 6: Diversification by Maximizing the Sharpe Ratio*
MPT is a mathematical formulation of diversification in investing, with the view of selecting a collection of investment assets that has collectively of lower risk than any individual asset. The diversification lowers risk even if the assets are positively correlated (Markowitz, 1959, 1952; Merton, 1972).

MPT models an asset's return as a stochastic function and defines risk as the standard deviation of return. MPT defines a portfolio as a weighted combination of assets, so that the return of a portfolio is the weighted combination of the assets' returns. By defining the

weights of different assets, MPT seeks to reduce the total variance of the portfolio return. A risk-free asset can be included in the portfolio, as well.

In 1966, William Forsyth Sharpe developed what is now known as the Sharpe Ratio. In Sharpe (1994), the Sharpe Ratio is defined as:

$$S = \frac{E[R_a - R_b]}{\sigma} = \frac{E[R_a - R_b]}{\sqrt{\text{var}[R_a - R_b]}}, \tag{71}$$

where R_a is the asset return, R_b is the return on a benchmark asset, such as the risk free rate of return or an index such as the S&P 500. $E[R_a - R_b]$ is the expected value of excess of the asset return over the benchmark return, and σ is the standard deviation of this expected excess return.

The influence of banks is directly included into the expressions of investors profit (14) and (28). Therefore, using the expressions (63) and (65), a sample estimate of the Sharpe Ratio of portfolio $P(x)$ can be expressed as

$$S(x) = \frac{m^u(x)}{s^u(x)}, \tag{72}$$

and the optimal portfolio x^S is defined by maximizing of $S(x)$

$$x^S = \arg\max_x S(x). \tag{73}$$

5. Results of Experiments

The general aim of the experiments is to indicate similarities and differences of the virtual and real stock markets. The specific aim is to evaluate the profitability of different investment strategies in both the virtual and real markets, which involve well-known companies. Important task is to explore the relation between the accuracy of prediction and the profits of ten different investment strategies. In the experiments, the profits and SE of eight players using different prediction models: AR(1), AR(3), AR(6), AR(9), AR-ABS(1), AR-ABS(3), AR-ABS(6), and AR-ABS(9) were tested.

Historical data was obtained automatically using the Yahoo data base. The historical prices of the following eight stocks were used: Microsoft (MSFT), Apple (AAPL), Google (GOOG), Nokia (NDK), Toyota (TM), Bank-of-America (BAC), Boeing (BA), and Nike (NKE). The time series start 1/30/2009 and end 12/17/2010.

Exploring the virtual data, the time period was 360 days (virtual working days). This represents approximately 18 months of real time. The average daily and final (at the end of tested period) values are estimated by 100 samples. The first four investment strategies were tested in the virtual environment including eight stocks of virtual companies.

The Table 1 shows average profits in real and virtual markets for different investment strategies and prediction methods at the end of time period. In real markets, the symbol R1 means the first strategy. In virtual markets, the first strategy is denoted as V1.

Table 1
The average of profits in virtual and real markets at the end of investment period.

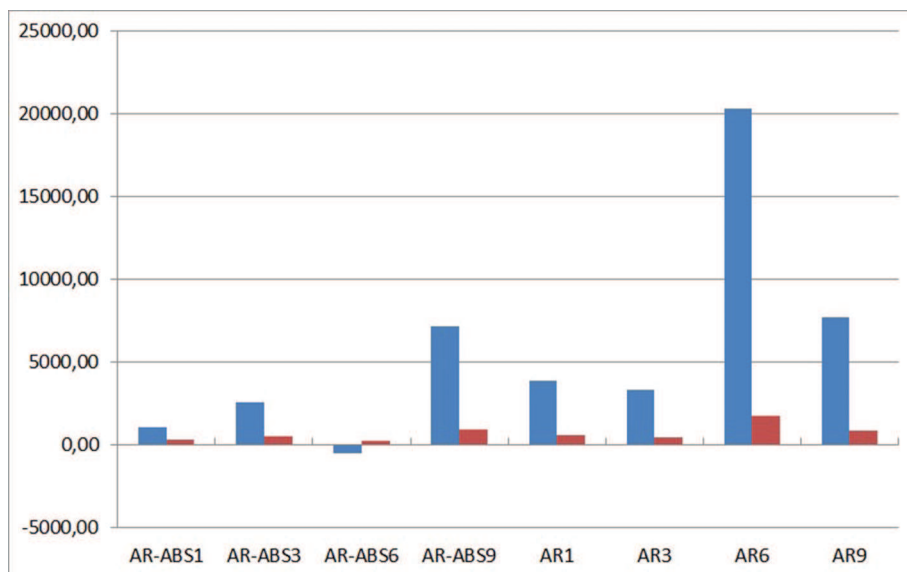| Strategy No.	Virtual market								
	AR-ABS1	AR-ABS3	AR-ABS6	AR-ABS9	AR1	AR3	AR6	AR9	Average
Real market									
R1	1059.36	2565.77	−518.07	7151.94	3864.07	3340.51	20258.2	7669.38	5673.89
R2	6181.79	−524.03	−2334.40	4908.25	5291.30	589.08	1962.95	525.94	2075.11
R3	1287.35	−431.64	−3013.07	−2459.46	42.58	−1169.67	8182.93	2665.21	638.03
R4	3806.22	3777.34	5629.49	7151.88	5579.59	4114.89	2456.41	4637.71	4644.19
R5	98.32	255.59	76.36	80.44	76.61	84.03	55.19	126.96	106.69
R6	104.74	172.13	106.81	147.21	78.54	155.51	76.37	169.17	126.31
R7	215.75	246.59	226.74	−151.71	275.74	182.30	456.44	−118.56	166.66
R8	157.95	189.20	125.61	187.59	175.38	99.35	502.59	158.26	199.49
R9	152.44	276.33	145.04	−19.42	292.92	55.86	178.46	−239.02	105.33
R10	212.88	217.46	224.92	127.27	153.77	154.98	278.39	73.39	180.38
Virtual market									
V1	3381.62	−3024.88	17577.00	−2795.87	739.39	1347.49	13689.31	833.36	3968.43
V2	−29.04	901.90	13596.29	2433.08	−388.54	1010.48	435.09	4590.43	2818.71
V3	−290.64	16527.23	4541.02	5683.21	−402.59	8273.67	39088.07	61273.55	16836.7
V4	−182.83	9262.32	3262.83	640.00	−311.73	−404.88	−202.49	−198.74	1483.06

Fig. 2. Average profits of eight prediction models using Strategy No. 1 in the real market.

In the real markets, the maximal profit was achieved using the strategy No. 1 by the prediction model AR(6). Figure 2 shows average profits of eight players using strategy No. 1 in the real market. In this and other average profit diagrams, the columns show the profits and the lengths of 95% confidence intervals (as the vertical dashes in the middle of columns). Here, the maximal profit was achieved using prediction model AR(6). In con-
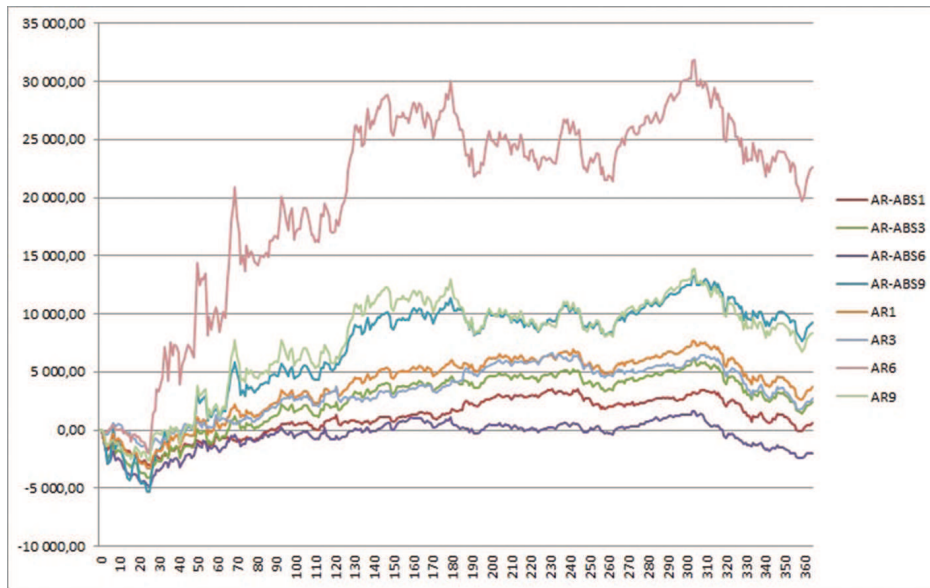
Fig. 3. Daily profits of eight prediction models in the real market.

trast, prediction model AR-ABS(6) provides some losses. This is unexpected result, since model AR-ABS(6) predicts asset prices considerably better, as compared with AR(6), see Fig. 5. In Fig. 3, the eight lines show the daily profits of eight players using strategy No. 1 in the real market

In thus figure, the graph of the best profits is similar to that of fastest growing stock rates. In the investigated period these were Bank-of-America (BAC) stocks recovering after the deep depression.

Figure 4 shows average profits of eight players using strategy No. 2 in the real market.

Using this trading rule, maximal profit was achieved by prediction model AR-ABS(1). Prediction model AR(6) which was the best using trading rule No. 1 did show rather poor results. This indicates that trading rules are at least as important for profits as the prediction models.

Figure 5 shows the estimates of the Standard Error (SE) and the Mean Absolute Error (MAE) of eight real stocks in 360 days.

Comparing errors in Figs. 5 with profits in Fig. 2, we see that the minimal prediction errors and the maximal profits both are achieved using the AR(1) model with minimal memory, this is in line with the Efficient Market Theory (Fama, 1995). However, comparing stock price prediction errors with profits in Fig. 4 we see different result where the greatest profit was provided by the model AR(6) with the memory parameter 6. Comparison of the prediction errors with profits by the both of trading rules Nos. 1 and 2, show that maximal profits are not necessarily achieved by the models with minimal price prediction errors

In Fig. 6, the most profitable portfolio in real stock market, defined using Strategy No. 1 and prediction model AR-ABS(6) is illustrated.
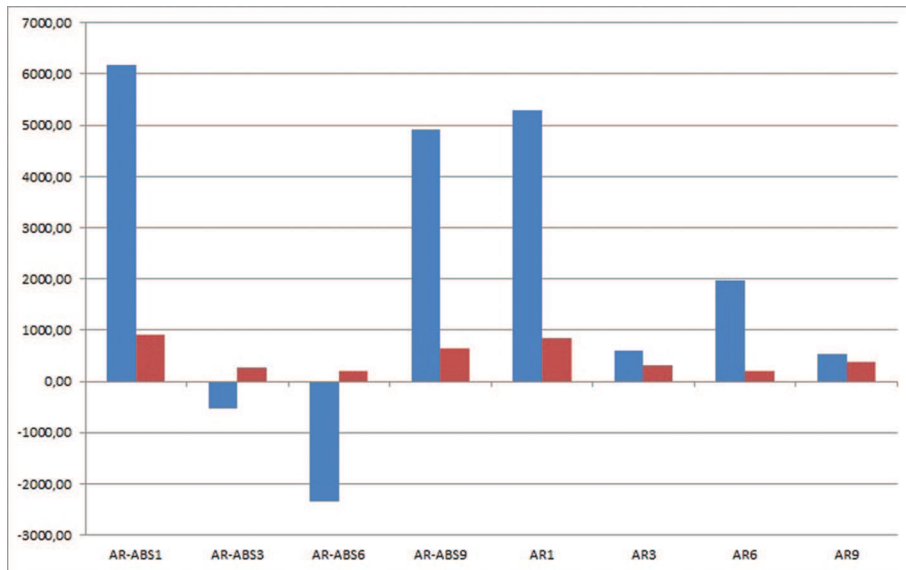
Fig. 4. Average profits of eight prediction models using Strategy No. 2 in the real market.
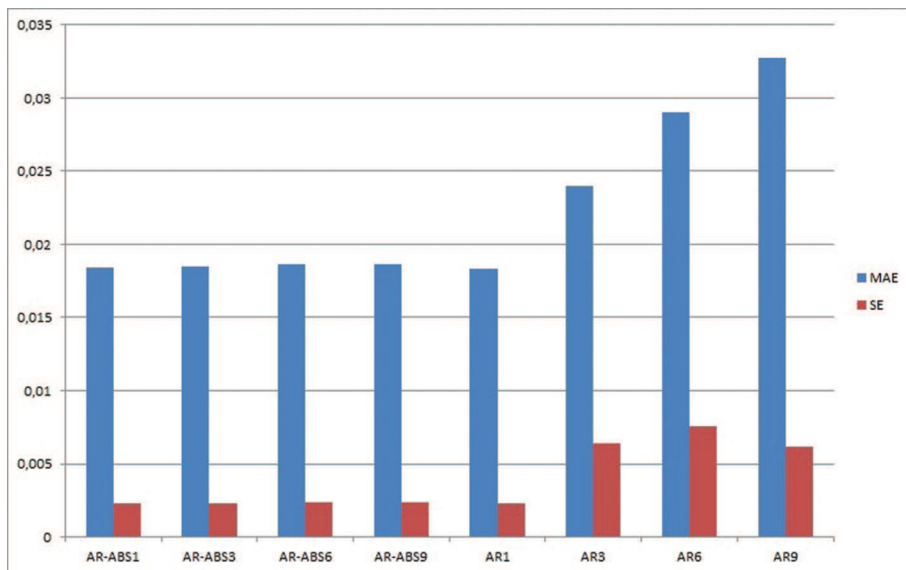

Fig. 5. SE and MAE of real stock market.

The dominant position of BAC stocks in the portfolio, is explained by the rapid recovery of these stocks after the depression.

In the virtual markets, the maximal profit was achieved using the strategy No. 3 by the prediction model AR(6). Figure 7 shows average profits of eight players using strategy No. 3 in the virtual market.
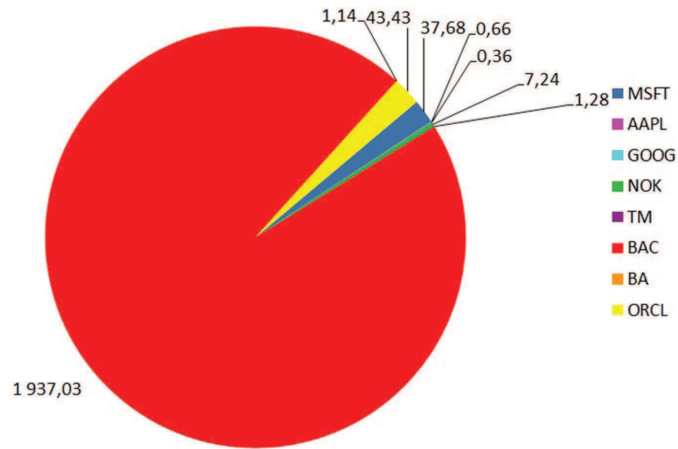
Fig. 6. The most profitable portfolio in real stock market, defined using Strategy No. 1 and prediction model AR-ABS(6).
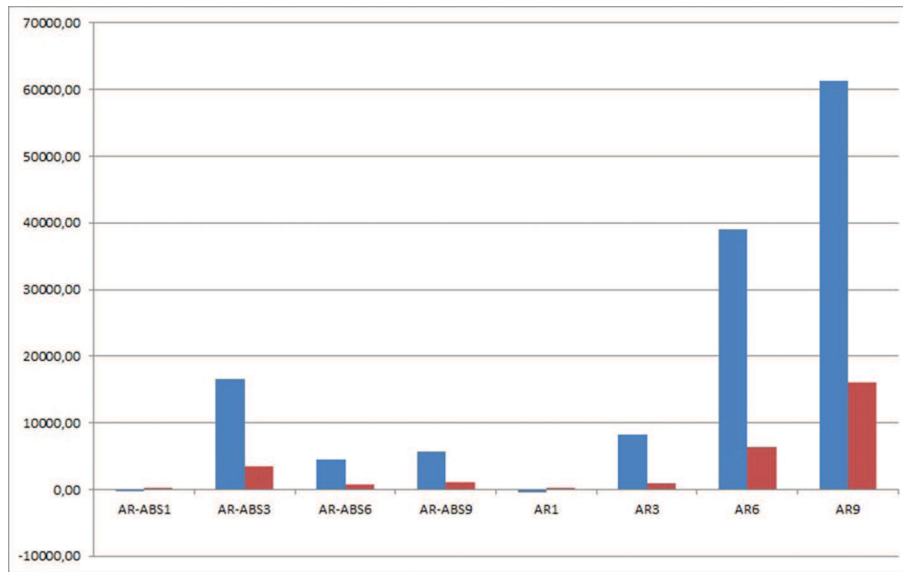

Fig. 7. Average profits of eight prediction models using Strategy No. 3 in the virtual market.

6. Software of the PORTFOLIO Model

The PORTFOLIO model is a part of the general on-line system for graduate studies and scientific collaboration (Mockus, 2006).

The initial web site (last modified in June, 2013) is at:
`http://optimum2.mii.lt.`

The mirror sites are at: `http://mockus.org/optimum`, `http://fmf.vgtu.lt/~mockus`, `http://pilis.if.ktu.lt/~jmockus`, `http://kopustas.elen.ktu.lt/~jmockus`, All the examples are in the form of Java applets and can be started by any browser with Java support (assuming that both Java and Javascript are enabled).

The PORTFOLIO model is in the web section "Global Optimization" and can be started by opening 'index.html' in the project "PORTFOLIO". The exception is Linux browsers which will be corrected in the future. The prediction model can be started by opening 'Starting original program' in the project "AR-ABS" of the same section.

In the Internet environment, we need platform-independent languages for running the software on remote computers. Java is more efficient for scientific calculations among such languages. Java applets provide a unique possibility for student-teacher interactions. Students can run the programs remotely and teachers can test students' results online.

7. Concluding Remarks

The Game Theory is a suitable framework to model financial markets because the future market price of financial assets depends on predictions (and subsequent actions) of the market participants with conflicting interests.

The idea that a financial market is a "casino" game was expressed in Keynes (2008). In Krugman (2000, 2008, 2009), the unpredictability of asset prices is explained by the irrational behavior of participants.

According to the model of this paper, the seemingly irrational behavior of market participants can be considered as rational under the specific conditions of decision making and the available information. From this standpoint, the differences between the Efficient Market Theory (Fama, 1995), assuming the rational behavior of market players and theories based on the irrational behavior (Krugman, 2000, 2008, 2009) are not so significant.

The proposed financial market model PORTFOLIO is designed as a tool for simulating market processes in response to different changes of market parameters and for estimating the expected profits of different investment strategies using both the historical and virtual data. Convenient user interactions are provided by implementing the model as a Java applet and publishing it in an open web-site (Mockus, 2013).

Since PORTFOLIO may be too simplistic for practical investing, it can serve as a useful tool for studies of market behavior by providing an easy way of simulating different scenarios of player strategies.

Thus, the PORTFOLIO model helps students of business informatics to understand better financial disasters that we are witnessing at present.

The new and unexpected result of experiments with the historical financial time series of the PORTFOLIO model is the observation that the minimal prediction errors do not provide the maximal profits. This is an important problem for further research.

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Apie investavimo strategijų optimizavimą virtualios finansų rinkos kontekste vertinant riziką individualiu požiūriu

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Optimalaus finansų investavimo (Portfelio) uždavinys buvo tiriamas pagrindinių finansinių organizacijų ir įžymių mokslininkų. Už modernios portfelio teorijos (MPT) sukūrimą buvo suteikta Nobelio premija. Šių darbų tikslas buvo sukurti priemones aktyvų įvairinimui priklausomai nuo priimtino rizikos lygio.

Skirtingai nuo MPT bei kitų investavimo modelių, šio darbo praktinis tikslas yra sukurti lengvai pritaikomą finansų rinkos modelį įvertinant individualius vartotojo poreikius bendros naudingumo teorijos kontekste. Šis tikslas yra naujas darbo elementas išskiriantis jį iš kitų panašių darbų nes čia optimizavimas vykdomas strategijų aibėje. Eksperimentai parodė, kad minimali prognozavimo paklaida nebūtinai maksimizuoja pelnus. Tai yra antras darbo naujumo elementas.